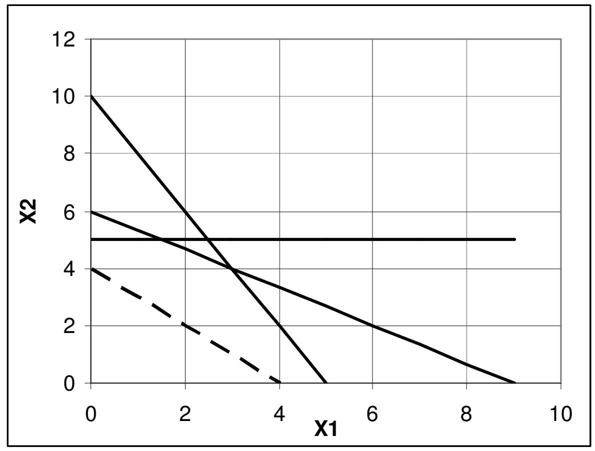
MATH 406 Test #1 Name________________________________
Fall 2004

1. (15 points) For the LP problem stated below use the graph provided to aid you in answering the questions.

Maximize $3x_1 + 3x_2$

s.t. $x_1, x_2 \geq 0$ and

$2x_1 + 3x_2 \leq 18$ (1st constraint)
$4x_1 + 2x_2 \leq 20$ (2nd constraint)
$x_2 \leq 5$ (3rd constraint)



a. (2 points) Shade the feasible region.

b. (2 points) Identify the optimal point (solution).

c. (2 points) Which constraints are binding?

d. (2 points) Which slack variables are zero in the optimal solution?

e. (2 points) What are the values of any non-zero slack variables in the optimal solution?

f. (3 points) Over what range can the objective function coefficient of x_2 vary before the current solution is no longer optimal?

g. (2 points) Which point is optimal if the objective function coefficient of x_2 is increased to 6?

2. (10 points) The only binding constraint in the LP problem is the second constraint.

$$
\begin{array}{lrcll}
\text{Max} & 2x_1 + x_2 & & & \\
\text{s.t.} & 2x_1 + 6x_2 & \leq & 300 & \text{(1}^{\text{st}}\text{ constraint)} \\
 & 2x_1 + 1x_2 & \leq & 10 & \text{(2}^{\text{nd}}\text{ constraint)} \\
 & 1x_2 & \leq & 45 & \text{(3}^{\text{rd}}\text{ constraint)} \\
 & x_1, x_2 & \geq & 0 &
\end{array}
$$

a. (1 point) What is the optimal solution?

b. (1 point) What is the value of the objective function at the optimal solution?

c. (4 points) Show how to determine the dual prices for the first and second constraints.

d. (2 points) Express this LP problem in standard form.

e. (2 points) Set up (but do not attempt to solve) the initial simplex tableau for this LP problem.

3. (10 points) Use the following Management Scientist output to answer the questions.

```
MAX 5X1+7X2
     S.T.
       1)   1X1<6
       2)   2X1+3X2<19
       3)   1X1+1X2<8

OPTIMAL SOLUTION:   Objective Function Value = 46.000

      Variable              Value             Reduced Costs
   --------------     ---------------      ------------------
         X1                   5.000                  0.000
         X2                   3.000                  0.000

     Constraint         Slack/Surplus           Dual Prices
   --------------     ---------------      ------------------
         1                    1.000                  0.000
         2                    0.000                  2.000
         3                    0.000                  1.000

OBJECTIVE COEFFICIENT RANGES

   Variable        Lower Limit        Current Value     Upper Limit
  ------------   ---------------    ---------------   ---------------
      X1                  4.667              5.000             7.000
      X2                  5.000              7.000             7.500

RIGHT HAND SIDE RANGES

  Constraint       Lower Limit        Current Value     Upper Limit
  ------------   ---------------    ---------------   ---------------
       1                  5.000              6.000   No Upper Limit
       2                 18.000             19.000            24.000
       3                  6.333              8.000             8.333
```

a. (2 points) Give the solution to the problem.

b. (2 points) Which constraints are binding?

d. (2 points) What would happen to the optimal value of the objective function if the right-hand side of constraint 1 were increased by 10?

e. (2 points) What would happen to the optimal value of the objective function if the right hand side of constraint 2 were increased by 3?

f. (2 points) What is the significance of the fact that the dual price for constraint 3 is 1?

4. (10 points) Provide the missing entries in the following partially completed simplex tableau (3 points).

Basis	c_B	x_1 3	x_2 4	x_3 5	s_1 0	s_2 0	
		1/2	1	0	1/2	-1/2	6
		0	0	1	-1/4	1	3
	z_j						
	$c_j - z_j$						

a. (1 point) What variables form the basis?

b. (1 point) What are the current values of the decision variables?

c. (1 points) What is the current value of the objective function?

d. (2 points) Which variable will be made positive next, and what will its value be?

e. (2 points) Which variable that is currently positive will become 0?

5. (5 points) A simplex tableau is shown below.

Basis	c_B	x_1 3	x_2 5	s_1 0	s_2 0	s_3 0	
s_1	0	1	0	1	0	0	4
x_2	5	0	1	0	1/2	0	6
s_3	0	3	0	0	1	1	6
	z_j	0	5	0	5/2	0	30
	$c_j - z_j$	3	0	0	-5/2	0	

a. (3 points) Do one more iteration of the simplex procedure.

Basis	c_B	x_1 3	x_2 5	s_1 0	s_2 0	s_3 0	
	z_j						
	$c_j - z_j$						

b. (1 point) After the iteration is performed what is the new complete solution?

c. (1 point) Is this new solution optimal?