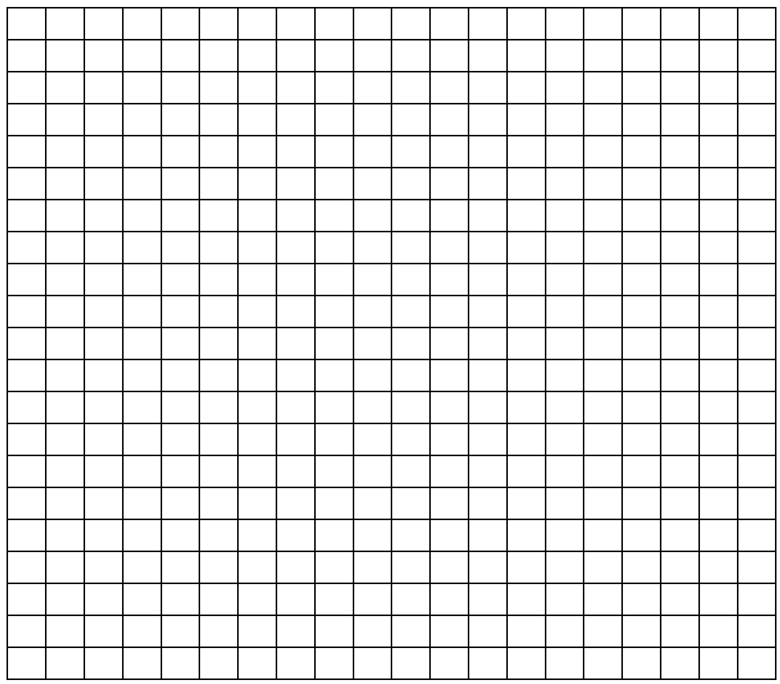
Session 13

1. Consider the following linear programming problem

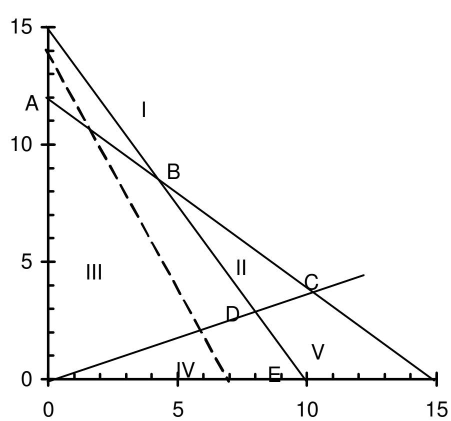
$$\begin{array}{lrcl} \text{Max} & 8X + 7Y & & \\ \text{s.t.} & 15X + 5Y & \leq & 75 \\ & 10X + 6Y & \leq & 60 \\ & X + Y & \leq & 8 \\ & X, Y & \geq & 0 \end{array}$$

a. Use a graph to show each constraint and the feasible region.
b. Identify the optimal solution point on your graph. What are the values of X and Y at the optimal solution?
c. What is the optimal value of the objective function?



2. Use this graph to answer the questions.

$$\begin{array}{ll} \text{Max} & 20X + 10Y \\ \text{s.t.} & \\ & 12X + 15Y \leq 180 \quad (1^{st} \text{ constraint}) \\ & 15X + 10Y \leq 150 \quad (2^{nd} \text{ constraint}) \\ & 3X - 8Y \leq 0 \quad (3^{rd} \text{ constraint}) \\ & X, Y \geq 0 \end{array}$$



a. Lightly shade the feasible region.
b. Which point (A, B, C, D, or E) is optimal?
c. Which constraints are binding?
d. If s_1 s_2, and s_3 the slack variables corresponding to constraints 1, 2, and 3 respectively,which slack variables are zero? What is the value of any non-zero slack variable?

3. The optimal solution of the linear programming problem is at the intersection of constraints 1 and 2.

$$
\begin{array}{ll}
\text{Max} & 2x_1 + x_2 \\
\text{s.t.} & 4x_1 + 1x_2 \leq 400 \\
 & 4x_1 + 3x_2 \leq 600 \\
 & 1x_1 + 2x_2 \leq 300 \\
 & x_1, x_2 \geq 0
\end{array}
$$

a. Over what range can the coefficient of x_1 vary before the current solution is no longer optimal?
b. Over what range can the coefficient of x_2 vary before the current solution is no longer optimal?
c. Compute the dual prices for the three constraints.

4. The following linear programming problem has been solved by The Management Scientist. Use the output to answer the questions.

LINEAR PROGRAMMING PROBLEM

MAX 25X1+30X2+15X3
S.T.

1) 4X1+5X2+8X3<1200
2) 9X1+15X2+3X3<1500

OPTIMAL SOLUTION Objective Function Value = 4700.000

Variable	Value	Reduced Costs
X1	140.000	0.000
X2	0.000	10.000
X3	80.000	0.000

Constraint	Slack/Surplus	Dual Prices
1	0.000	1.000
2	0.000	2.333

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X1	19.286	25.000	45.000
X2	No Lower Limit	30.000	40.000
X3	8.333	15.000	50.000

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	666.667	1200.000	4000.000
2	450.000	1500.000	2700.000

a. Give the complete optimal solution.
b. Which constraints are binding?
c. What is the dual price for the second constraint? What interpretation does this have?
d. Over what range can the objective function coefficient of x_2 vary before a new solution point becomes optimal?
e. By how much can the amount of resource 2 decrease before the dual price will change?
f. What would happen if the first constraint's right-hand side increased by 700 and the second's decreased by 350?

5. Set up the initial simplex tableau for the LP problem of exercise #3 above.

6. A portion of a simplex tableau is

Basis	c_B	x_1 20
x_2	25	.2
s_2	0	3
	z_j	5
	$c_j - z_j$	15

Give a complete explanation of the meaning of the $z_1 = 5$ value as it relates to x_2 and s_2.

7. Given the following initial simplex tableau

Basis	c_B	x_1 5	x_2 8	x_3 12	s_1 0	s_2 0	s_3 0	
s_1	0	3	4	5	1	0	0	80
s_2	0	9	15	20	0	1	0	250
s_3	0	1	-1	2	0	0	0	20
	z_j	0	0	0	0	0	0	0
	$c_j - z_j$	5	8	12	0	0	0	

a. What variables form the basis?
b. What are the current values of the decision variables?
c. What is the current value of the objective function?
d. Which variable will be made positive next, and what will its value be?
e. Which variable that is currently positive will become 0?
f. What value will the objective function have next?

8. A simplex tableau is shown below.

Basis	c_B	x_1 3	x_2 5	x_3 8	s_1 0	s_2 0	s_3 0	
s_1	0	3	6	0	1	0	-9	126
s_2	0	-5/2	-1/2	0	0	1	-9/2	45
x_3	8	1/2	1/2	1	0	0	1/2	18
	z_j	4	4	8	0	0	4	144
	$c_j - z_j$	-1	1	0	0	0	-4	

a. Do one more iteration of the simplex procedure.
b. Subsequent to that iteration, what is the current complete solution?
c. Is this solution optimal? Why or why not?