# Session 16 – Example

**Write the following LP problem in standard form.**

**Maximize $1x_1 + 3x_2$**

**s.t.** $x_1, x_2 \geq 0$ **and**

$3x_1 + 2x_2 \geq 12$
$1x_1 + 2x_2 = 8$

**Introducing surplus variable $s_1$ and artificial variables $a_1$ and $a_2$ we obtain**

**Maximize $1x_1 + 3x_2 + 0s_1 - Ma_1 - Ma_2$**

**s.t.** $x_1, x_2, s_1, a_1, a_2 \geq 0$ **and**

$3x_1 + 2x_2 - 1s_1 + 1a_1 = 12$
$1x_1 + 2x_2 + 1a_2 = 8$

**Initial Simplex Tableau:**

| | | $x_1$ | $x_2$ | $s_1$ | $a_1$ | $a_2$ | |
|---|---|---|---|---|---|---|---|
| ***Basis*** | $c_B$ | 1 | 3 | 0 | -M | -M | |
| $a_1$ | -M | 3 | 2 | -1 | 1 | 0 | 12 |
| $a_2$ | -M | 1 | 2 | 0 | 0 | 1 | 8 |
| $z_j$ | | -4M | -4M | M | -M | -M | |
| $c_j - z_j$ | | 1+4M | 3+4M | -M | 0 | 0 | -20M |

**On the 1st iteration $x_2$ will enter the basis and $a_2$ will depart:**

| | | $x_1$ | $x_2$ | $s_1$ | $a_1$ | |
|---|---|---|---|---|---|---|
| ***Basis*** | $c_B$ | 1 | 3 | 0 | -M | |
| $a_1$ | -M | 2 | 0 | -1 | 1 | 4 |
| $x_2$ | 3 | 1/2 | 1 | 0 | 0 | 4 |
| $z_j$ | | 3/2-2M | 3 | M | -M | 12-4M |
| $c_j - z_j$ | | -1/2+2M | 0 | -M | 0 | |

**On the 2nd iteration _____ will enter the basis and _____ will depart.**

| | | $x_1$ | $x_2$ | $s_1$ | |
|---|---|---|---|---|---|
| ***Basis*** | $c_B$ | 1 | 3 | 0 | |
| | | | | | |
| $x_2$ | 3 | | | | |
| $z_j$ | | | | | |
| $c_j - z_j$ | | | | | |

**Optimal Solution:**

**Consider the following minimization problem.**

**Minimize $3x_1 + 2x_2$**
**s.t. $x_1, x_2 \geq 0$ and**

**$3x_1 + 2x_2 \geq 12$**
**$1x_1 + 2x_2 = 8$**

**Converting to a maximization problem we obtain**

**Maximize $-3x_1 - 2x_2$**
**s.t. $x_1, x_2 \geq 0$ and**

**$3x_1 + 2x_2 \geq 12$**
**$1x_1 + 2x_2 = 8$**

**Introducing surplus variable $s_1$ and artificial variables $a_1$ and $a_2$ we obtain**

**Maximize $-3x_1 - 2x_2 + 0s_1 - Ma_1 - Ma_2$**
**s.t. $x_1, x_2, s_1, a_1, a_2 \geq 0$ and**

**$3x_1 + 2x_2 - 1s_1 + 1a_1 = 12$**
**$1x_1 + 2x_2 + 1a_2 = 8$**

**Initial Simplex Tableau:**

| | | $x_1$ | $x_2$ | $s_1$ | $a_1$ | $a_2$ | |
|---|---|---|---|---|---|---|---|
| *Basis* | $c_B$ | -3 | -2 | 0 | -M | -M | |
| $a_1$ | -M | 3 | 2 | -1 | 1 | 0 | 12 |
| $a_2$ | -M | 1 | 2 | 0 | 0 | 1 | 8 |
| $z_j$ | | -4M | -4M | M | -M | -M | |
| $c_j - z_j$ | | -3+4M | -2+4M | -M | 0 | 0 | -20M |

**On the 1st iteration $x_2$ will enter the basis and $a_2$ will depart:**

| | | $x_1$ | $x_2$ | $s_1$ | $a_1$ | |
|---|---|---|---|---|---|---|
| *Basis* | $c_B$ | -3 | -2 | 0 | -M | |
| $a_1$ | -M | 2 | 0 | -1 | 1 | 4 |
| $x_2$ | -2 | 1/2 | 1 | 0 | 0 | 4 |
| $z_j$ | | -1-2M | -2 | M | -M | -8-4M |
| $c_j - z_j$ | | -2+2M | 0 | -M | 0 | |

**On the 2nd iteration $x_1$ will enter the basis and $a_1$ will depart.**

| | | $x_1$ | $x_2$ | $s_1$ | |
|---|---|---|---|---|---|
| *Basis* | $c_B$ | -3 | -2 | 0 | |
| $x_1$ | -3 | 1 | 0 | -1/2 | 2 |
| $x_2$ | -2 | 0 | 1 | 1/4 | 3 |
| $z_j$ | | -3 | -2 | 1 | -12 |
| $c_j - z_j$ | | 0 | 0 | -1 | |

**Optimal Solution:**