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Directions. Work on this test without the aid of any other person, and provide no aid to any other person. You may refer to your notes, textbook, or any other books you care to consult. Attach this cover sheet to your solutions and submit it with your completed test on later than 10:00 am on Monday, December 6. Be sure to cite and credit any references you use.

Work these problems in the spirit of the approach to problem solving used in this course. Pay particular attention to clearly communicating your assumptions, thoughts and problem solving processes and to justifying your procedures and conclusions. This is an opportunity to show off what you have learned and how you can apply that knowledge.

Sign one of the following two statements.
Statement: I worked this test in compliance with the above directions.
(Signature)
Statement: I am unable to sign the statement above due to the exception(s) listed below.
(Signature)

1. Job Assignment Problem (20 points)

XYZ Corp. has $n$ applicants $\left(A_{1}, A_{2}, \ldots, A_{n}\right)$ for $m$ jobs $\left(J_{1}, J_{2}, \ldots, J_{m}\right)$. Each applicant is rated for each job on a scale ranging from 0 to 5 . Develop a model for assigning applicants to jobs. Apply your model to the special case described by the rating matrix $\mathrm{R}=\left(\mathrm{r}_{\mathrm{ik}}\right)$ which is shown below. In each case $r_{i k}$ represents applicant $A_{i}$ 's rating (qualifications) for job $J_{k}$. Thus, $r_{i k}=3$ would mean that applicant $A_{i}$ is rated 3 for job $J_{k}$. Given four applicants for four jobs where each applicants rating for each job is as shown in the rating matrix $R$, which applicants should be assigned to which jobs? (Is your best assignment of applicants to jobs unique?)

$\mathrm{R}=$|  | $\mathbf{J}_{1}$ | $\mathbf{J}_{2}$ | $\mathbf{J}_{3}$ | $\mathrm{~J}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{1}$ | 0 | 1 | 0 | 1 |
| $\mathrm{~A}_{2}$ | 3 | 3 | 5 | 0 |
| $\mathrm{~A}_{3}$ | 2 | 1 | 1 | 1 |
| $\mathrm{~A}_{4}$ | 3 | 2 | 3 | 2 |

## 2. A Problem of Resource Allocation (20 points)

In a region of West Texas, the vegetation is easily modified by grazing animals from mixed vegetation to a dominance of one or two types. In this area, it is common to see herds of cattle, flocks of sheep, and flocks of mohair goats in pastures. Whitetail deer and wild turkey are common if there is enough browse and mixed vegetation. Catfish will thrive in ponds if there is suitable vegetation cover to prevent siltation. Ranchers sell beef, wool, mutton, and mohair, and lease rights to deer and turkey hunting and catfishing. The relative monetary income values per animal are: cattle, 10; goats, 1 ; sheep, 1 (wool and mutton combined); deer, 0.5 ; turkey, 0.05 ; and fish, 0.001.

Suppose a rancher owns 10 sections of such land, which has a maximum carrying capacity of 10 animal units per section per year. The animal unit equivalents for the various species per animal are: cattle, 1 ; sheep, .2 ; goats, .25 ; deer, .3 ; and essentially zero for turkeys and fish. To properly organize his operation for livestock production, the rancher must have at least 20 cattle and at least 20 goats on his ranch, and he also wants to have some sheep and deer. He can maintain the desired vegetation cover for turkeys and fish if he has (a) cattle, sheep, goats, and deer, (b) cattle, sheep, and goats, or (c) cattle, goats, and deer, but no more than $75 \%$ of the grazing load (measured in animal units) may be due to just cattle and goats. Of course, he wants the total number of animals to be equal to or less than the carrying capacity of the range. Furthermore, he can put in no more than one pond per section, each of which will support at most 500 fish. He can "harvest" no more than $25 \%$ of the catfish in each pond per year. The requirements and habits of the wild turkey are such that he cannot maintain more than 2 flocks ( 10 birds per flock) per section, and he cannot harvest more than $20 \%$ of the turkey population on his ranch per year. He keeps only castrated male goats for mohair, and shears the goats once each year. The cattle, sheep, and deer harvests which will maintain a given population (constant) are respectively about $25 \%$, $35 \%$, and $15 \%$ of each population per year. For simplification, the $35 \%$ for sheep includes both wool and mutton.

How many individuals of each species should the rancher have in order to maximize annual profit?

