Solutions for Practice Exercises

1. a. x_2, x_3
   b. $x_1 = 0, x_2 = 6, x_3 = 3$
   c. 39
   d. x_1, 12
   e. x_2
   f. 51

2. a. $x_1 = 0, x_2 = 0, x_3 = 8, s_1 = 4, s_2 = 0, s_3 = 10, Z = 64$
   b. If we make one unit of x_1 it will require .4 units of s_1, at no cost, .8 units of x_3 production will have to be foregone at a cost of 8 each, and .8 units of s_3 will be needed at no cost. In order to make a unit of x_1, profit of 32/5 will be given up. (Equivalent profit contribution is 32/5)
   c. The profit for a unit of x_2 is 4. However, in order to make a unit of x_2, profit of 32/5 would be given up as resources are diverted. The net improvement is a loss, shown by the -12/5. Therefore it makes no sense to produce x_2. (actual profit contribution – equivalent profit contribution = -12/5)

3. Since non-basic variable s_3 has a relative profit of zero, this means that any increase in s_3 will produce no change in the objective function value. Thus, since s_3 can be made a basic variable, the resulting basic feasible solution will also have an optimum value of 14. An alternative optimal solution is indicated whenever there exists a non-basic variable whose relative profit ($c_j - z$) row coefficient is zero in the optimal solution

4. Max $x_1 + 2x_2 - Ma_1 - Ma_2$

   s.t. $3x_1 + 4x_2 + s_1 = 100$
   $2x_1 + 3.5x_2 - s_2 + a_1 = 60$
   $2x_1 - 1x_2 + a_2 = 50$

   $x_1 , x_2 , s_1 , s_2 , a_1 , a_2 \geq 0$

   The initial basis would include s_1, a_1, and a_2 .

5. The tableau indicates an infeasible solution.

6.

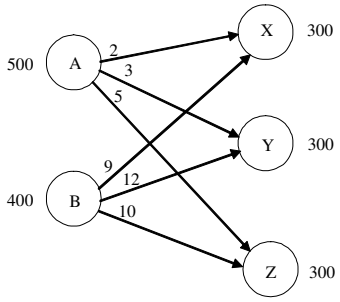
Basis	c_B	x_1 100	x_2 120	x_3 85	s_1 0	s_2 0	
s_1	0	3	1	6	1	0	120
s_2	0	5	8	2	0	1	160
	z_j	0	0	0	0	0	0
	$c_j - z_j$	100	120	85	0	0	

Basis	c_B	x_1 100	x_2 120	x_3 85	s_1 0	s_2 0	
s_1	0	2.375	0	5.75	1	-.125	100
x_2	120	.625	1	.25	0	.125	20
	z_j	75	120	30	0	15	2400
	$c_j - z_j$	25	0	55	0	-15	

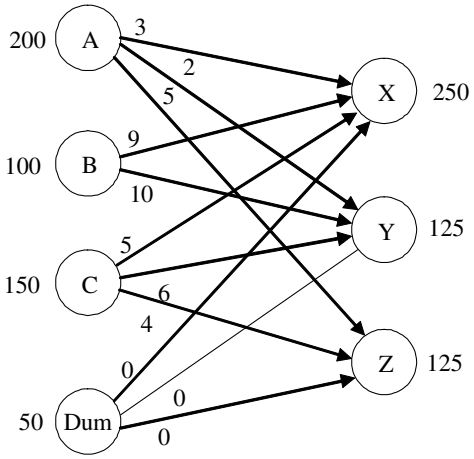
Basis	c_B	x_1 100	x_2 120	x_3 85	s_1 0	s_2 0	
x_3	85	.413	0	1	.174	-.0217	17.391
x_2	120	.522	1	0	-.043	.1304	15.652
	z_j	97.745	120	85	9.63	13.8035	3356.52
	$c_j - z_j$	2.283	0	0	-9.565	-13.8035	

Basis	c_B	x_1 100	x_2 120	x_3 85	s_1 0	s_2 0	
x_3	85	0	-.792	1	.2083	-.125	5
x_1	100	1	1.917	0	-.0833	.250	30
	z_j	100	124.38	85	9.3755	14.375	3425
	$c_j - z_j$	0	-4.375	0	-9.375	-14.375	

8.



9.



Min $3X_{AX}\ 2X_{AY} + 5X_{AZ} + 9X_{BX} + 10X_{BY} + 5X_{CX} + 6X_{CY} + 4X_{CZ}$

s.t.
$$X_{AX} + X_{AY} + X_{AZ} \le 200$$
$$X_{BX} + X_{BY} \le 100$$
$$X_{CX} + X_{CY} + X_{CZ} \le 150$$
$$X_{DX} + X_{DY} + X_{DZ} \le 50$$
$$X_{AX} + X_{BX} + X_{CX} + X_{DX} = 250$$
$$X_{AY} + X_{BY} + X_{CY} + X_{DY} = 125$$
$$X_{AZ} + X_{BZ} + X_{CZ} + X_{DZ} = 125$$
$$X_{ij} \ge 0$$

10. Ship 10 from St. Louis to Indianapolis, 45 from St. Louis to Ft. Wayne, 45 from St. Louis to South Bend, 100 from Evansville to Terre Haute, 50 from Bloomington to Terre Haute, and 50 from Bloomington to Indianapolis. The total cost is 1755.

11. Min $3x_{16} + 2x_{14} + 3x_{15} + 5x_{24} + 6x_{25} + 2x_{32} + 8x_{34} + 10x_{35} + 5x_{46} + 9x_{47} + 12x_{56} + 15x_{57}$

s.t.
$$x_{16} + x_{14} + x_{35} \le 500$$
$$x_{24} + x_{25} - x_{23} \le 400$$
$$x_{32} + x_{34} + x_{35} \le 300$$
$$x_{46} + x_{47} - (x_{14} + x_{24} + x_{34}) = 0$$
$$x_{56} + x_{57} - (x_{15} + x_{25} + x_{35}) = 0$$
$$x_{16} + x_{46} + x_{56} = 600$$
$$x_{56} + x_{57} = 600$$