Example of an axiom system, Σ_1

undefined terms: player, team, contains (contained in)

A_1: Every team contains at least two players

A_2: There exists at least three players

A_3: Given any two distinct players P_1 and P_2, there exists a unique team containing them.

A_4: Given any team T and a player P not contained in T, then there exists a unique team T' containing P where there is no player contained in both T and T'.

Models for Σ_1?

(see p. 17)

Some theory

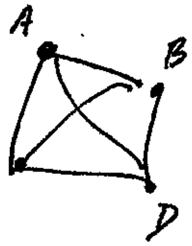
Theorem 1.1 If there are two distinct teams then there are three distinct teams.

(What does it mean to say two teams are distinct?)

Theorem 1.2 If there are three distinct teams, then there are four distinct teams.

Models for Σ

#1 { player $\longleftrightarrow$ usual real-world meaning
team $\longleftrightarrow$ a set of players (non-empty)
contains $\longleftrightarrow$ set membership

#2 {


player $\longleftrightarrow$ vertex
team $\longleftrightarrow$ edge
contains $\longleftrightarrow$ graph theory context

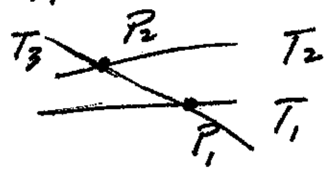
#3 { player $\longleftrightarrow$ point
team $\longleftrightarrow$ line
contains $\longleftrightarrow$ incident

#4 {


Consistency
law of contradiction
thms in Σ true in each logical model

Theorem 1.1 If there are two distinct teams, then there are three distinct teams.

Proof. Suppose there are two distinct teams T_1 and T_2. By A_1 & A_2 there must be a player P_1 in T_1 that is not in T_2 and there must be a player P_2 in T_2 that is not in T_1

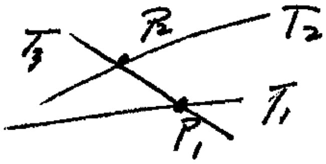


By A_3 there is a unique team T_3 containing P_1 and P_2. T_3 cannot be T_1 since P_2 is not in T_1 and T_3 cannot be T_2 since P_1 is not in T_2. So, T_3 is a third team distinct from T_1 and T_2. Consequently, if there are two distinct teams, then there are three distinct teams.

Theorem in Σ_1

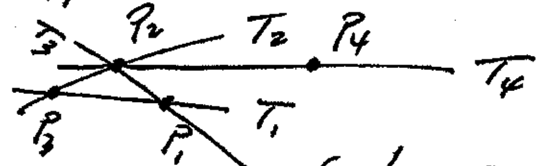
If there are three distinct teams, then there are four distinct players

Proof Consider the distinct players P_1, P_2, P_3 and distinct teams T_1, T_2, T_3 in the proof of Theorem 1.1.



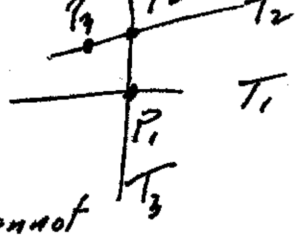
There exists a player P_3 other than P_2 on T_2 by A_1 and A_2. We consider two cases.

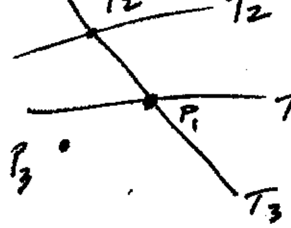
Case 1 P_3 is in T_1



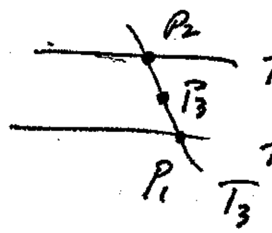
In this case, by A_4 there must be a fourth distinct tree T_4 that contains P_2 but contains no player in T_1. T_4 must contain a fourth distinct player P_4 by A_1 and A_3. (We know we can choose $P_4 \neq P_2$ by A_1 and clearly $P_4 \neq P_1$ and $P_4 \neq P_3$ by $A3$). So, in this case we have four distinct players

Case 2 P_3 is not in T_1





In this case there must be a fourth player P_4 on T_1 by A_1. Of course P_4 cannot be either of P_1, P_2, P_3.



Consequently, if there are 3 distinct teams there are 4 distinct players