Introduction to Difference Equations

1. Consider the sequence: 2, 4, 6, 8, …

a. The first term is _____. We will denote this by $a_1 = 2$.
b. The second term is _____. We will denote this by a_2 = _____.
c. The third term is _____. We will denote this by a_3 = _____.
d. The fourth term is the third term plus _____. We write $a_4 = a_3$ + _____.
So, a_4 = _____.
e. The fifth term is the fourth term plus _____. We write $a_5 = a_4$ + _____.
So, a_5 = _____.
f. $a_6 = a_5$ + _____ = _____.
g. The next term in the sequence is always the current term plus _____.
k. The (n+1)st term is the nth term plus _____.
l. $a_{n+1} = a_n$ + _____. *(This is called a difference equation.)*

This sequence can be defined by recursion via a difference equation:

$a_1 = 2$ and,
$a_{n+1} = a_n + 2$ for $n \geq 1$.

This sequence can also be defined explicitly by the functional equation

$a_n = 2n$.

2. Consider the sequence in the following table.

N	1	2	3	4	5	6	7
b_n	3	8	13	18	23		

a. Complete the table.

b. Define the sequence recursively using a difference equation.

c. Define the sequence explicitly using a functional equation.

3. Consider the sequence in the following table. Here we start with n = 0.

n	0	1	2	3	4	5	6	7	8
c_n	5	7	13	23	37	55			

a. Complete the table.

b. Define the sequence recursively using a difference equation.

c. Define the sequence explicitly using a functional equation.

4. Consider the sequence in the following table.

n	0	1	2	3	4	5	6	7	8
d_n	32	40	50	62.5	78.1	97.7			

a. Assume that the values of d_n are rounded to the nearest 0.1 and complete the table.

b. Define the sequence recursively using a difference equation.

c. Define the sequence explicitly using a functional equation.