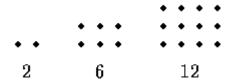# Figurate Numbers

Figurate numbers can be represented by dots arranged in the shape of certain geometric figures. For example, the 1st three *rectangular numbers* are shown below.
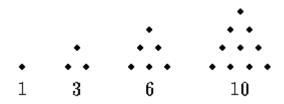
1.   Consider the sequence of *rectangular numbers* whose 1st three terms are shown below.

Find the next three rectangular numbers by drawing the corresponding arrays. Let R_n denote the nth rectangular number. Complete the table below and find R_{10} and R_{20}. Try to find a rule for finding R_n for any value of n.

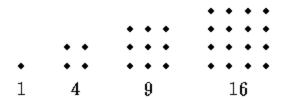| n	1	2	3	4	5	6	7	8	9	10	11
R_n											

2. The 1st four *triangular numbers* are shown below.

Determine the next three triangular numbers by drawing the corresponding trianglular arrays. Let T_n denote the nth triangular number. Complete the table below and find T_{10} and T_{20}. Try to find a rule for finding T_n for any value of n.

n	1	2	3	4	5	6	7	8	9	10	11
T_n											

3.   The 1st four *square numbers* are shown below.

$$
\begin{array}{cccc}
& & & \bullet\ \bullet\ \bullet\ \bullet \\
& & \bullet\ \bullet\ \bullet & \bullet\ \bullet\ \bullet\ \bullet \\
& \bullet\ \bullet & \bullet\ \bullet\ \bullet & \bullet\ \bullet\ \bullet\ \bullet \\
\bullet & \bullet\ \bullet & \bullet\ \bullet\ \bullet & \bullet\ \bullet\ \bullet\ \bullet \\
1 & 4 & 9 & 16
\end{array}
$$

Suppose we denote the nth square number by S_n. Write a rule for determining S_n for any n.

To the ancient Greeks, the square root of a number in this sequence was the number of dots along one side of the square that represents the number. For non-square natural numbers, they used a clever technique to estimate the square roots. This technique is illustrated below.

$$
\sqrt{11} \approx 3\tfrac{2}{7} \qquad\qquad \sqrt{8} \approx 2\tfrac{4}{5}
$$

Use this technique to arrive at the approximation $[\sqrt{22}] \approx 4\tfrac{6}{9}$.