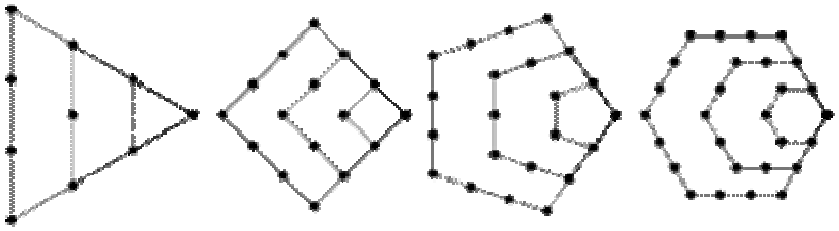
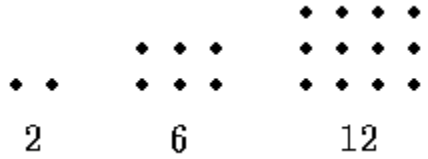
Figurate Numbers

Figurate numbers can be represented by dots arranged in the shape of certain geometric figures.



1. Consider the sequence of ***rectangular numbers*** whose 1st three terms are shown below.



Find the next three rectangular numbers by drawing the corresponding arrays. Let R_n denote the n^{th} rectangular number. Complete the table below and find R_{10} and R_{20}. Use the method of finite differences and determine rules for finding R_n for any value of n.

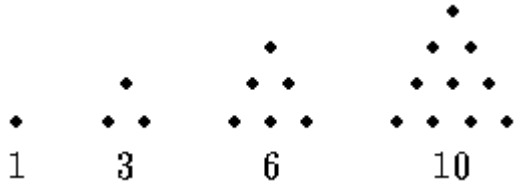
n		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$\mathbf{R_n}$												
$\mathbf{\Delta R_n}$												
$\mathbf{\Delta\Delta R_n}$												

Verbal Rule:

Difference Equation:

Functional Equation:

2. The 1st four ***triangular numbers*** are shown below.



Determine the next three triangular numbers by drawing the corresponding triangular arrays. Let T_n denote the n^{th} triangular number. Complete the table below and find T_{10} and T_{20}. Use the method of finite differences and determine rules for finding T_n for any value of n.

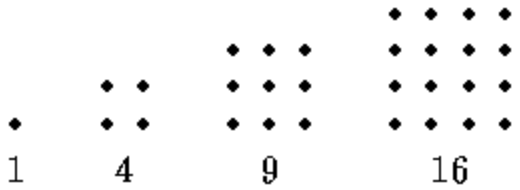
n		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$\mathbf{T_n}$												
$\mathbf{\Delta T_n}$												
$\mathbf{\Delta\Delta T_n}$												

Verbal Rule:

Difference Equation:

Functional Equation:

3. The 1st four ***square numbers*** are shown below.



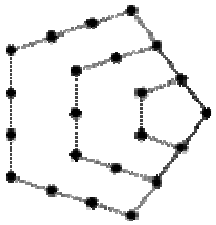
Suppose we denote the n^{th} square number by S_n. Write rules for determining S_n for any n.

Verbal Rule:

Difference Equation:

Functional Equation:

4. We now consider the sequence of ***pentagonal numbers***.



Suppose we denote the n^{th} pentagonal number by P_n. Suppose we denote the n^{th} square number by S_n. Write rules for determining S_n for any n.

n		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$\mathbf{P_n}$												
$\mathbf{\Delta P_n}$												
$\mathbf{\Delta\Delta P_n}$												

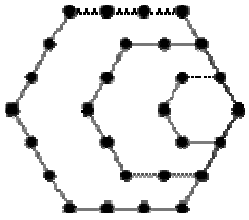
Verbal Rule:

Difference Equation:

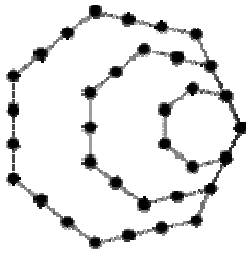
Functional Equation:

5. Investigate the sequences of hexagonal, heptagonal, and octagonal numbers in a manner like we have been investigating the previous figurate numbers. Can you determine a general rule for finding the n^{th} k-gonal number?

Hexagonal



Heptagonal



Octagonal