

Sequences, Method of Finite Differences, Method of Ratios, and Functional Equations

-A Working Backwards Approach-

Sequences Generated by Polynomial Functions

Linear: $S_n = an + b$

n	0	1	2	3
S_n	b	a+b	2a+b	3a+b
ΔS_n		a	a	a

Quadratic: $S_n = an^2 + bn + c$

n	0	1	2	3	4
S_n	c	a+b+c	4a+2b+c	9a+3b+c	16a+4b+c
ΔS_n		a+b	3a+b	5a+b	7a+b
$\Delta\Delta S_n$			2a	2a	2a

Cubic: $S_n = an^3 + bn^2 + cn + d$

n	0	1	2	3	4
S_n	d	a+b+c+d	8a+4b+2c+d	27a+9b+3c+d	64a+16b+4c+d
ΔS_n		a+b+c	7a+3b+c	19a+5b+c	37a+7b+c
$\Delta\Delta S_n$			6a+2b	12a+2b	18a+2b
$\Delta\Delta\Delta S_n$				6a	6a

Sequences Generated by Exponential Functions:

Exponential: $S_n = ar^n$

n	0	1	2	3
S_n	a	ar	ar²	ar³
S_{n+1}/S_n		r	r	r

Exercises: Find explicit functional equations for each sequence.

1. 1, 6, 15, 28, 45, 66, ...
2. 16, 24, 36, 54, 81, 121.5, ...
3. 3, 7, 11, 15, 19, 23, ...
4. 3, 10, 30, 76, 160, 294, ...