## MATH 230 Symmetries of a Square

How many ways can you place your 2 " by 2 " square piece of paper into a 2 " by 2 " frame? Label the corners of your square $1,2,3,4$ in either a clockwise or counterclockwise manner. Turn the paper square over and label the opposite side so that each corner is associated with the same number on both sides. That is, the corner labeled " 1 " on one side is labeled " 1 " on the reverse side as well; the corner labeled " 2 " on one side is labeled " 2 " on the reverse side as well, etc. Use your labeling scheme to show how many different ways the square may be fitted into a 2 " by 2 " frame. Use the blank frames below to display your solution.


Consider and illustrate the effect of each of the following transformations on a square.
1.

Rotate $\mathbf{9 0}^{\mathbf{0}}$ counterclockwise
$\left(\mathbf{R}_{90}\right)$
2. Rotate $\mathbf{1 8 0}^{\circ}$ counterclockwise ( $\mathbf{R}_{180}$ )

3. Rotate $270^{\circ}$ counterclockwise ( $\mathbf{R}_{270}$ )
5. Flip around diagonal \#1( $\left.\mathrm{D}_{1}\right)$
7.

4. Rotate $360^{\circ}$ counterclockwise ( $\mathbf{R}_{360}$ )


Flip around diagonal \#2 ( $\mathrm{D}_{2}$ )
8.



Composition of Transformations - Consider what single transformation is equivalent to a sequence of two transformations. Suppose we first perform the transformation in the left most columm and then perform the transformation in the top row. What single transformation has the same effect on a square?

|  | $\mathbf{R}_{360}$ | $\mathbf{R}_{90}$ | $\mathbf{R}_{180}$ | $\mathbf{R}_{270}$ | $\mathbf{D}_{1}$ | $\mathbf{D}_{\mathbf{2}}$ | $\mathbf{H}$ | $\mathbf{V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{R}_{360}$ |  |  |  |  |  |  |  |  |
| $\mathbf{R}_{90}$ |  |  |  |  |  |  |  |  |
| $\mathbf{R}_{180}$ |  |  |  |  |  |  |  |  |
| $\mathbf{R}_{270}$ |  |  |  |  |  |  |  |  |
| $\mathbf{D}_{\mathbf{1}}$ |  |  |  |  |  |  |  |  |
| $\mathbf{D}_{\mathbf{2}}$ |  |  |  |  |  |  |  |  |
| $\mathbf{H}$ |  |  |  |  |  |  |  |  |
| $\mathbf{V}$ |  |  |  |  |  |  |  |  |

