Problem What is the relationship between the number of teams in a round-robin tournament and the number of games that must be played?

Understand the Problem

In a round-robin tournament each team must play each of the other teams exactly once. I must be careful to count each game and count no game twice.

I will develop a rule for calculating the number of games that must be played given any number of teams in the tournament.

Make a Plan

I will consider the special cases of 2, 3, 4, 5, 6, ... teams in the tournament and determine the number of games that must be played in each case. I will record my results in a table and I will look for a pattern that I can use to determine my rule. I will represent teams by letters A, B, C, etc, and games will be denoted by pairs of those letters. "AB" will stand for the game between teams

A and B. Note that "AB" and "BA" represent the same game.

Carry Out the Plan

✓ Two Teams: A, B
One Game: AB

✓ Three Teams: A, B, C
Three Games: AB, AC, BC

✓ Four Teams: A, B, C, D
Six Games: AB, AC, AD, BC, BD, CD

✓ Five Teams: A, B, C, D, E
Ten Games: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

Number of Teams	Number of Games	change in Number of Games
2	1	
		2
3	3	
		3
4	6	
		4
5	10	
		5
6	15	

I predict there will be 15 games for 6 teams

Six Teams: A, B, C, D, E, F
Fifteen Games: AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF

It looks like a tournament with 7 teams will have six more games than a tournament with 6 teams.

So, far our table is as follows:

# of teams	# of Games	Change in # of games
2	1	
3	3	2
4	6	3
5	10	4
6	15	5
7	21	6
8	28	7
9	36	8
10	45	9

If we let N = the number of teams and G_N = the number of games for a tournament with N teams it looks like

$$(\star)\begin{cases} G_2 = 1 \\ G_N = G_{N-1} + (N-1) \end{cases}$$

That is, the number of games for N teams is the number of games for $(N-1)$ teams increased by $(N-1)$.

We can extend our table to any number of teams using the pattern I have found.

<u>Look Back</u>

Using our rule, the number of games for 11 teams G_{11} is given by $G_{11} = G_{10} + (10)$ which is $G_{11} = 45+9 = 54$. which checks when the table is extended.

The rule (A) that I found is a difference equation. Perhaps I can find a corresponding explicit functional representation.

Given N teams, each team plays (N-1) others. So, each of the N teams plays (N-1) games. That looks like N(N-1) games but each game involves two teams, so, the actual number of games should be $\frac{N(N-1)}{2}$. Let's make a table using that rule.

N	$\frac{N(N-1)}{2}$
2	$1 = \frac{2(1)}{2}$
3	$3 = \frac{3(2)}{2}$
4	$6 = \frac{4(3)}{2}$
5	$10 = \frac{5(4)}{2}$
6	$15 = \frac{6(5)}{2}$

So, another rule is

$$G_N = \frac{N(N-1)}{2}.$$