Chris,

It looks like you are taking a productive route to finding a solution. In the case where n = 8, I think there are actually a few more triangles in the picture. You might check your addition. Some tips follow your notes below. Nice observations. Do you get the same pattern if n = 7? If not, you might look at the cases where n is even and then the cases where n is odd.

Chris' notes:

Gen Eq (right side up)	Σ{(1+2+3+…+n),(1+2+3+…+(n-1)),(1+2+3+…+(n-2)),…,(1+2),(1)}
Gen Eq (up side down)	Σ{(1+2+3+…+(n-1)),(1+2+3+…+(n-3)),(1+2+3+…+(n-5)),…,(1+2+3),(1)}
Gen Eq (total)	Σ{Gen Eq (right side up), Gen Eq (up side down)}

- No addend of the Sigmas may be negative

When n=8 (the one you handed out), the answer is 160 triangles.

Further suggestions/tips.

Case n = 8.

Size of sub-triangle	Number found in figure (# with points up) + (#with points down)
1x1x1	(1+2+…+8) + (1+2+…+7) = 64
2x2x2	(1+2+…+7) + (1+2+…+5) = 43
3x3x3	(1+2+…+6) + (1+2+3) = 27
4x4x4	(1+2+…+5) + 1 = 16
5x5x5	(1+2+3+4) = 10
6x6x6	(1+2+3) = 6
7x7x7	(1+2) = 3
8x8x8	(1) = 1

So, in the case where n = 8, there are 170 triangles in the figure.

It looks like Chris has applied inductive reasoning using this case to conjecture a general solution. We might now try looking at those cases where n = 1, 2, 3, 4, 5, 6, 7 and see if we can capture the relevant relationship with an explicit formula. (Might we deduce a function T where T(n) denotes the number of triangles in an nxnxn triangular array? We have seen that T(8) = 170.)

Using Chris' approach on those cases where n is even we get: T(2) = 5, T(4) = 27, T(6) = 78, T(8) = 170, T(10) = 315, T(12) = 525. In conversation, Chris suggested looking at differences to see if we could get some hint regarding properties of the function T we seek.

Consider the table below.

	2		4		6		8		10		12	
	5		27		78		170		315		525	
Δ		22		51		92		145		210		
Δ^2			29		41		53		65			
Δ^3				12		12		12				

Applying inductive reasoning, our third differences seem to be constant (not proved). Maybe T is a cubic polynomial (thrid derivative a constant). If so, we seek a, b, c, and d such that $T(n) = an^3 + bn^2 + cn + d$.

We believe that T(2) = 5, T(4) = 27, T(6) = 78, and T(8) = 170. We might even assume that T(0) = 0. If T(0) is 0, then d must be 0 – symplifying our problem!

Using what we believe to be true,

$T(0) = 0 = a(0)^3 + b(0)^2 + c(0) + d$

$T(2) = 5 = a(2)^3 + b(2)^2 + c(2) + d$

$T(4) = 27 = a(4)^3 + b(4)^2 + c(4) + d$

$T(6) = 78 = a(6)^3 + b(6)^2 + c(6) + d$

We have a system of equations. From the first equation, d = 0. Hence we can reduce our problem to three equations.

$8a + 4b + 2c = 5$
$64a + 16b + 4c = 27$
$216a + 36b + 6c = 78$

Eliminating c from the last two equations we get

$48a + 8b = 17$
$192a + 24b = 63$

Eliminating b from the third equation,

$48a = 12$.

Letting a = 2/8 and back substituting we get b = 5/8 an c = 2/8.

Maybe for n even, $T(n) = [2n^3 + 5n^2 + 2n]/8$ or $T(n) = [n(2n+1)(n+2)]/8$.

Check it out! But does this function work for n odd?