Middle School Modeling Activities Linking Mathematics and Science

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at

The National Council of Teachers of Mathematics Southern Regional Conference Charleston, SC November 8, 2003

Abstract

We illustrate some ways to use science laboratory activities to facilitate and enrich the teaching of mathematical concepts. Through experimentation and exploration, we apply mathematics to analyze real-world phenomena.

The activities illustrated are included in a workshop course intended for in-service middle school teachers. The course, titled "Mathematical Models and Modeling for Teachers," was developed under the auspices of a Salisbury University, NSF-funded, program identified as the Allied Delmarva Enhancement Program for Teachers (ADEPT). For additional information about this course see

http://faculty.salisbury.edu/~dccathca/ModelsWorkshop/description.html

Our Approach to Modeling from Data

Given a real world phenomenon to consider, we ask our students to demonstrate the following steps in fitting a model to data relative to the phenomenon:

- Pre Activity
  - Formulate the key problem or question (show you understand the problem).
  - Communicate preconceptions.
  - Discuss the limitations, assumptions, and scope of the proposed model.
- Activity and Post Activity
  - Collect and organize data.
  - Analyze and interpret data.
  - Fit an appropriate model. Vary parameters and test goodness of fit by an identified numerical criterion (sum of errors, average error, percent error).
  - Discuss the limitations, assumptions, and scope of the model.
  - Summarize and report findings. If possible, identify a proportionality relationship to validate your choice of models.
  - Revisit preconceptions, reflect, describe, formulate, evaluate, support, generalize, research, and suggest.

Breaking Strength of Spaghetti Bridges

Guided Group Discussion (Approximately 20 minutes)

How is the length of a bridge related to the strength of the bridge? To address this question we build spaghetti bridges and measure the strength of each bridge by the number of pennies required to break the spaghetti.

Goals of discussion:

- Identify issues, variables, and parameters.
- Identify possible representations and preconceptions of the problem.
- Refine the problem.

Questions to Address in groups:

- What physical principles affect the relationship?
- What other factors or variables might come into play?
- How can we represent these relationships? (Discuss not only the *type* of representation, but also the qualitative properties of the relationship.)

Collecting Data (20 Minutes)

Length	Breaking Weight
2.0”	
2.5”	
3.0”	
3.5”	
4.0”	
4.5”	
5.0”	

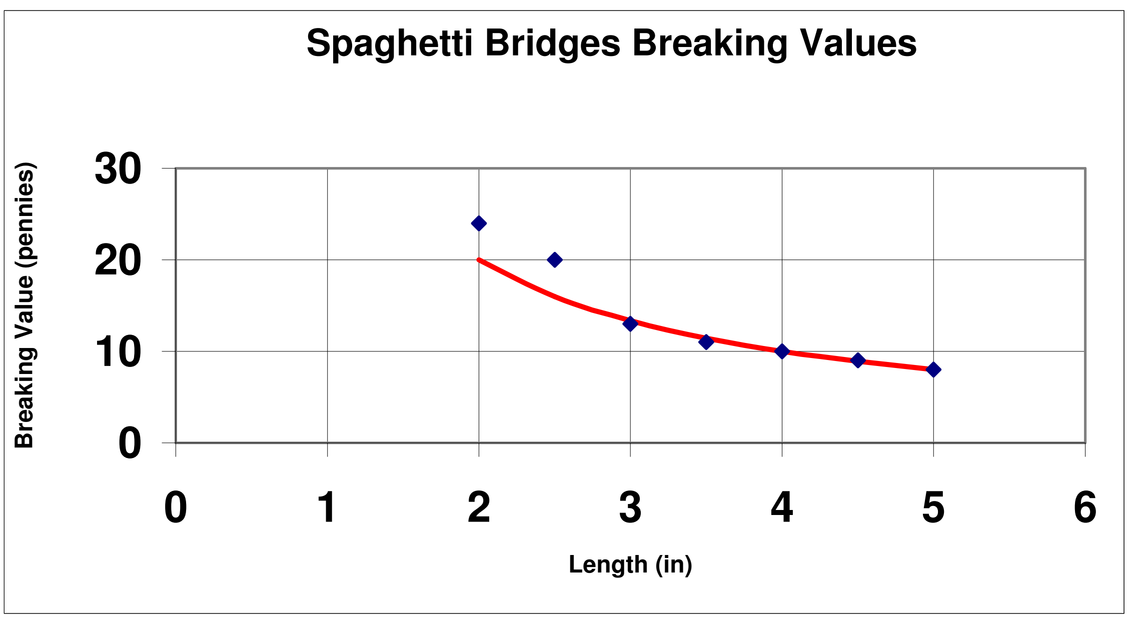
Plotting Data (20 Minutes)

- Plot the data on graph paper (W vs L) and sketch a line that fits this data. Estimate the slope & intercept of line and interpret these values.
- Transform the data. Instead of graphing W vs. L, graph W vs. a function of L; e.g. L^n. Preconceptions often suggest a useful function to try. Fit a straight line to the result; interpret its slope and intercept.
- Make predictions from these lines of the number of pennies required to break strands of lengths 6", 4.75", 2.25", and 1".
- Compare reliability, or confidence in predictive power, for the two lines.

Sample Data on Spaghetti Bridges

Bridge Length **L** ***(in)***	***Breaking Value*** **B** ***(pennies)***	**L x B**	***Model Value*** **B = K / L**	***Error***
2.0	*24*	*48.0*	*20*	*4.0*
2.5	*20*	*50.0*	*16*	*4.0*
3.0	*13*	*39.0*	*13*	*0.3*
3.5	*11*	*38.5*	*11*	*0.4*
4.0	*10*	*40.0*	*10*	*0.0*
4.5	*9*	*40.5*	*9*	*0.1*
5.0	*8*	*40.0*	*8*	*0.0*

Best Guess for K: 40 **Sum of errors : 8.8**



The Draining Bottle

Goal: To discover a relationship between the height of water in a bottle and the rate at which the height of the water changes as the bottle is drained through a hole in the bottom. Think of water draining from a bathtub.

Equipment
- Clear two-liter soda bottle
- Nail
- Tape
- Ruler
- Watch with second hand
- Basins or bags to catch draining water (or go outside)

Procedure
- Punch a hole in the bottle with the nail about 5 centimeters from the bottom of the clear two-liter soda bottle.
- Tape the ruler vertically to the side of the bottle so that the 0 centimeter mark is aligned with the hole punched in the bottle.
- First person puts finger over the hole and fills bottle with water to a height of about 15 centimeters
- First person calls out as finger is removed from hole and calls out height of the water in whole centimeters as the water level passes that height
- Second person calls out elapsed time each time the level passes a height
- Third person records results in the table.

Elapsed Time T	Height of H_2O h	Δh	Δt	$\Delta h/\Delta t$

The rate at which the height is changing is the change in height, Δh, divided by the change in time, Δt.

Any curve we fit to the data should have the property that when h is 0, $\Delta h/\Delta t$ is also 0 (convince yourself by thinking about the water draining from the bottle). In particular, if we fit a line, its equation will be

$$\Delta h/\Delta t = m * h;$$

i.e., the intercept is 0. This means when sketching a line on a graph, one edge of the ruler is on (0,0). (If using a graphing calculator or a spreadsheet consult the reference manual to determine how to set the intercept to 0.)

Fit a line to the data. (A straight line with intercept 0 does not fit the data very well.)

A little research on the Web or a careful look at the data itself might suggest a power function; i.e.

$$\Delta h/\Delta t = m * h^p$$

Use Excel or your graphing calculator to fit this curve. Note the values of m and p

<u>Reflect on Preconceptions.</u>

Where have we confirmed our conceptions? Modified our conceptions?

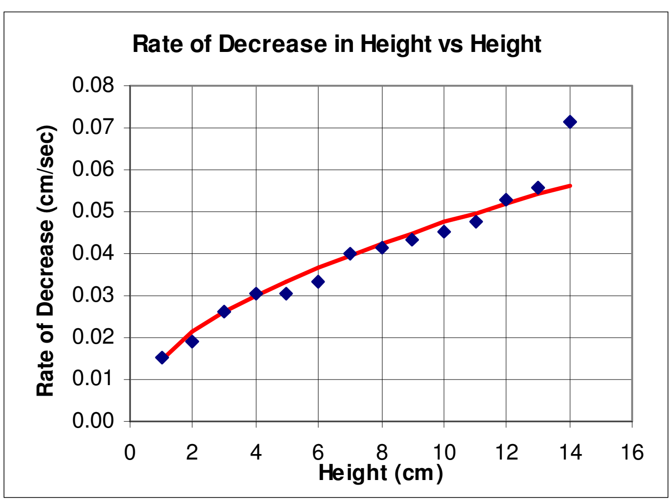
Sample Data for the Draining Bottle

Hole size – approximately 2.5 mm

What is the relationship between the height of the water and the rate at which the height is changing (decreasing)?

Time (t) (sec)	Height (h) (cm)	Avg. Rate of Decrease in h w.r.t t (cm/sec)	Square Root of Height	Ratio of Rate to Sq Root	Model Value (cm/sec)
0	15		3.87		
14	14	0.07	3.74	0.019	0.06
32	13	0.06	3.61	0.015	0.05
51	12	0.05	3.46	0.015	0.05
72	11	0.05	3.32	0.014	0.05
94	10	0.05	3.16	0.014	0.05
117	9	0.04	3.00	0.014	0.05
141	8	0.04	2.83	0.015	0.04
166	7	0.04	2.65	0.015	0.04
196	6	0.03	2.45	0.014	0.04
229	5	0.03	2.24	0.014	0.03
262	4	0.03	2.00	0.015	0.03
300	3	0.03	1.73	0.015	0.03
352	2	0.02	1.41	0.014	0.02
418	1	0.02	1.00	0.015	0.02

Best guess: 0.015 **r = 0.015** ($\sqrt{h}$)



Bouncing Golf Ball – a CBL/CBR Activity

We use a TI 82 (or TI 73) calculator and a calculator-based ranger (CBR) to help students develop and evaluate a mathematical model to describe the motion of bouncing golf ball. The following questions are posed:

- How high will the ball bounce on the n^{th} bounce?
- What is the ball's height at any time between two specified successive bounces? (Perhaps between the first and second or between second and third time it hits the floor?)
- What is the ball's velocity at any time between two specified successive bounces?
- How will an individual ball's characteristics affect our results?

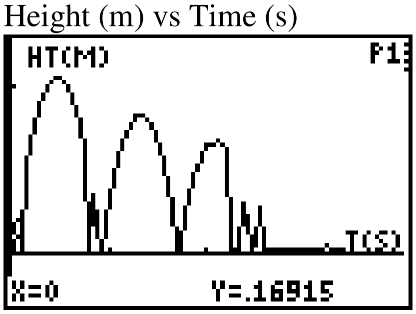
In the pre activity students communicate their preconceptions by sketching graphs predicting the following relationships, and by explaining in words the nature of the anticipated relationships.

- The maximum height of the ball as a function of the bounce number.
- The maximum height of the ball as a function of the previous maximum height.
- The height of the ball above the floor as a function of time since it was dropped.
- The ball's velocity as a function of time since it was dropped.
- The ball's acceleration as a function of time since it was dropped.

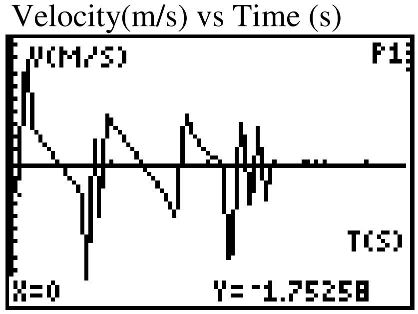
We also discuss physical factors that need to be considered as they address the questions for this activity. In particular, which to consider and which to ignore.

Here are sample height-time and velocity-time graphs from a TI 82 LCD screen.

Height (m) vs Time (s)



Velocity(m/s) vs Time (s)



(For sample instructions and materials to gather and display the data relevant to this activity, see the exploration described on the TI web site, www.ti.com\calc\docs\act\hsmotion08.htm.)

We use the TI's trace feature to determine the times at which the ball hits the ground the second and third times, to find the time the ball reaches its maximum height on the second bounce and to determine the ball's height at those three times.

Let t = time elapsed, in seconds, and h(t) = ball's height, in meters, at time t.

Using the trace function on the sample we found the ball's maximum heights on three successive bounces are

(*) h(0.47) ≈ 1.05, h(1.33) ≈ 0.83, and h(2.11) ≈ 0.67.

Let b(n) = the maximum height, in meters, achieved on the nth bounce and note that

b(2)/b(1) • 0.8, b(3)/b(2) • 0.8, and b(4)/b(3) • 0.8.

Using the data in (*) above, we hope students conjecture that the ball's maximum height on any bounce is approximately 0.8 its maximum height on the previous bounce. If b(n) denotes the maximum height of the n^{th} bounce, then for each bounce after the first, b(n) = 0.8b(n-1). Or, in general $b(n) = (0.8)^{n-1}b(1)$. Of course, students must gather data on more bounces to validate this model.

We can also estimate that

(**) h(0.95) ≈ 0.0, h(1.33) ≈ 0.83, and h(1.72) ≈ 0.0.

Using qualitative and quantitative properties of the Height vs Time graph and model fitting heuristics we have developed previously, students are likely to conjecture a quadratic relationship exists between the ball's height and time since a bounce started. (We do not allow students to use the TI's curve fitting routines during model formulation.) So, the students can fit a quadratic relationship to the second bounce using either the points established in (**) above or data stored in their TI 82.

Students would derive an approximate relationship close to

(★) $h(t) \approx -5.6(t - 1.33)^2 + 0.83 \approx -5.6t^2 + 14.9t - 9.08$, for $0.95 \leq t \leq 1.72$

for the second bounce.
With a little internet research, it is possible to find that scientists would expect that the coefficient of t^2 is –4.9. That fact could lead to some interesting discussions during the model validation step. (We have found that students (and their instructors) do not always generate "good" data.)

Visiting the Velocity vs Time graph we use the TI 82's trace feature to examine the ball's velocity between 0.95 sec and 1.72 sec.

We let t = time elapsed, in seconds, and v(t) = velocity, in meters per second, of the ball at time t.

The best we can do in this case is

(***) v(0.99) ≈ 3.73, v(1.33) ≈ 0.16, v(1.38) ≈ -0.31, v(1.68) ≈ -3.99.

In the case, students would probably conjecture a linear relationship between the ball's velocity and the time since a bounce started. Students would derive an approximate relationship close to

(★★) $v(t) \approx -11.2t + 14.82$, for $0.99 \leq t \leq 1.68$

From calculus (or a little internet research) we could expect the that $v(t)$ should be approximately equal to $-11.2t + 14.9$, so in this case we did very well.

Discussion: How does our approach to this activity differ from the approach typically found in published versions?

Additional Information

Web Site for the mathematical modeling workshop for middle school teachers:

http://faculty.salisbury.edu/~dccathca/ModelsWorkshop/description.html

Web sites for two other mathematical modeling courses:

A course for prospective elementary school teachers-

http://faculty.salisbury.edu/~dccathca/MATH115/abstract.htm

A course for upper level mathematics majors-

http://faculty.salisbury.edu/~dccathca/MATH465/Syllabus.htm