## The Draining Bottle

Goal: To discover a relationship between the height of water in a bottle and the rate at which the height of the water changes as the bottle is drained through a hole in the bottom. Think of water draining from a bathtub.

## Equipment

- Clear two-liter soda bottle
- Nail
- Tape
- Ruler
- Watch with second hand
- Basins or bags to catch draining water (or go outside)


## Procedure

- Punch a hole in the bottle with the nail about 5 centimeters from the bottom of the clear two-liter soda bottle.
- Tape the ruler vertically to the side of the bottle so that the 0 centimeter mark is aligned with the hole punched in the bottle.
- First person puts finger over the hole and fills bottle with water to a height of about 15 centimeters
- First person calls out as finger is removed from hole and calls out height of the water in whole centimeters as the water level passes that height
- Second person calls out elapsed time each time the level passes a height
- Third person records results in the table.

| Elapsed Time T | $\begin{aligned} & \text { Height of } \mathrm{H}_{2} \mathrm{O} \\ & \mathrm{~h} \end{aligned}$ | $\Delta \mathrm{h}$ | $\Delta \mathrm{t}$ | $\Delta \mathrm{h} / \Delta \mathrm{t}$ |
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The rate at which the height is changing is the change in height, $\Delta \mathrm{h}$, divided by the change in time, $\Delta \mathrm{t}$.

Any curve we fit to the data should have the property that when $h$ is $0, \Delta h / \Delta t$ is also 0 (convince yourself by thinking about the water draining from the bottle). In particular, if we fit a line, its equation will be

$$
\Delta \mathrm{h} / \Delta \mathrm{t}=\mathrm{m} * \mathrm{~h} ;
$$

i.e., the intercept is 0 . This means when sketching a line on a graph, one edge of the ruler is on $(0,0)$. (If using a graphing calculator or a spreadsheet consult the reference manual to determine how to set the intercept to 0 .)

Fit a line to the data. (A straight line with intercept 0 does not fit the data very well.)
A little research on the Web or a careful look at the data itself might suggest a power function; i.e.

$$
\Delta \mathrm{h} / \Delta \mathrm{t}=\mathrm{m} * \mathrm{~h}^{\mathrm{p}}
$$

Use Excel or your graphing calculator to fit this curve. Note the values of $m$ and $p$

## Reflect on Preconceptions.

Where have we confirmed our conceptions? Modified our conceptions?

Sample Data for the Draining Bottle
Hole size - approximately 2.5 mm

What is the relationship between the height of the water and the rate at which the height is changing (decreasing)?

|  |  | Avg. Rate <br> of Decrease | Square Root <br> of | Ratio of Rate <br> to Sq Root | Model <br> Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Time (t) | Height (h) <br> in h w.r.t t <br> $(\mathbf{c m})$ | Height |  |  |  |
| 0 | 15 |  |  |  | $(\mathrm{~cm} / \mathrm{sec})$ |

Best guess: $0.015 \quad \mathbf{r}=\mathbf{0 . 0 1 5}(\sqrt{h})$


