The CBL and CBR in a Modeling Course for Middle School Teachers

Presented by:

Don Cathcart (dccathcart@salisbury.edu)
Steve Hetzler (smhetzler@salisbury.edu)
Bob Tardiff (rmtardiff@salisbury.edu)

Department of Mathematics & Computer Science
Salisbury University
Salisbury, MD 21801

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Abstract

We share some ways to employ TI's calculator-based laboratory (CBL) and calculator-based ranger (CBR) in science activities designed to motivate and enrich mathematical concepts and explore connections between math and science. The activities illustrated are included in a workshop course intended for in-service middle school teachers. The course, titled "Mathematical Models and Modeling for Teachers," was developed under the auspices of a Salisbury University, NSF-funded, program identified as the Allied Delmarva Enhancement Program for Teachers (ADEPT).

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-Session Handouts-

What is Math ADEPT?

Math ADEPT (Allied Delmarva Enhancement Program for Teachers) is a unified program of interconnected courses in mathematics at Salisbury University (SU), offered for graduate credit, and developed to meet the content and pedagogical needs of in-service middle school teachers across Delmarva (MD, DE, VA). Salisbury University has received a 3-year grant from the National Science Foundation (NSF), which will help fund the development of the ADEPT program. Teachers participating in ADEPT will acquire a deep, conceptual understanding of the mathematics content of standards-based middle school curriculum within a classroom environment, which embraces discovery-based learning within small classes. ADEPT will also promote a pedagogically sound transfer of participants' content mastery into their middle school classrooms and will enhance teachers' mathematical confidence and attitudes. ADEPT participants will create, implement, modify, and improve lesson plans under the guidance of ADEPT faculty, mentor teacher colleagues, and other school system partners. Key Features of ADEPT courses:

- Courses to meet the needs of elementary-certified teachers taking the "middle school challenge"
- A challenging yet supportive atmosphere to help you gain a higher level of content mastery
- Coverage of the content that is hands-on and involves active learning (problem-solving, group work, written assignments, technology, oral presentations, etc).
- In most ADEPT courses, there are no tests, with grades based on written assignments, projects, assignments to modify/improve lesson plans, class presentations, class participation, etc.
- While ADEPT courses are not methods courses (with one exception) coverage of content is integrated with activities applicable to middle school classes; NCTM readings, model curricula, manipulatives, and technology will also be integrated.
- Opportunities to interact with teachers from other counties, and who teach at grade levels ranging from elementary to high school, but who are interested in middle school math.
- Free of tuition For the 6 ADEPT courses below; NSF is covering the tuition.
- Participants who complete 4 or more ADEPT courses will receive a Certificate of Recognition, a 1-year membership in NCTM including a subscription to the journal *Mathematics Teaching in the Middle Schools*, a $100 stipend, plus a copy of NCTM's Standards 2000. Recognizing the significant level of achievement ADEPT courses represent, ADEPT instructors may also write letters to the school systems of ADEPT participating teachers.

The ADEPT Courses:

- Conceptual Algebra for Teachers
- Number Theory from a Historical and Multi-cultural Perspective
- Geometry: From Euclid to Modern Day
- Data Analysis
- Mathematical Modeling for Middle School Teachers
- Middle School Mathematics in a Teaching Context with Instructional Technology

SU has recently received funding from MHEC/Eisenhower for two more courses:

- Mathematical Reasoning and Discrete Mathematics, and
- The Cartesian Triad: Algebra, Geometry, and Coordinates in the Plane

The eight ADEPT courses (7 mathematics and 1 methodology), are in line with the recommendations of the Maryland Math Commission (MMC). We hope the eight courses will eventually lead to a certification in middle school mathematics and perhaps as elementary mathematics specialist.

The Modeling Course (Summer 2002 Version)

MATH 506 Selected Topics: Mathematical Models and Modeling for Teachers

Instructors: Drs. Robert M. Tardiff, Donald C. Cathcart, and Steven M. Hetzler

Objectives: This course is designed to help you discover and express mathematical relationships found in the world around you. As you work through the course activities, we hope that you will

- see connections between mathematics and other areas,
- become adept in using some technological tools such as calculators, computers, microcomputer- and calculator-based laboratories (MBL's and CBL's),
- develop a variety of problem-solving strategies,
- gain success in solving non-routine problems, and become skillful in explaining and justifying your reasoning,
- adapt course activities for use in your middle school lessons, and
- develop a conceptual framework for presenting mathematical models and applications at the middle school level.

The schedule: The course will be delivered as a series of ten mini-workshops: 9:00 – 12:00 and 1:00 – 4:00, M – F, July 29th – August 2nd, 2002, followed by two days of student presentations and peer mentoring: Monday and Tuesday, August 12th and 13th, 2002.

Schedule for ADEPT Modeling Course for Teachers (Summer 2002)

Day 1, AM:	Introduction & Housekeeping Details **Pendulum Activity** – Modeling Process (Activity #10 in text) Activity #1 in text & Activity #2 in text Revisit Pendulum Data Round-Robin Tournaments (Activity #7 in text) Mathematical Representations (graphs, diagrams, equations, matrices) Difference & Functional Equations Some Types of Change – Examples (1^{st} & 2^{nd} differences; ave. rate of change, % change)

Day 1, PM:	Global Positioning System (GPS) & Modeling
	Model Fitting Problems:
	    Peg Game
	    Tower Puzzle (Activity #8 in text)
	Difference Equations & Functional Equations

Day 2, AM:	Building Bridges – Breaking Points The Mathematical Modeling Process Calculators & Spreadsheets Curve Fitting – Better Fit Criteria Surface Areas of Colored Rods – Activity #6 in text
Day 2, PM:	Draining the Bathtub Problem Stella Model for Bathtub Problem Modeling with Stella Calculators & Spreadsheets (Day 1&2 data sets + other data sets as able)
Day 3, AM:	**Newton's Law of Cooling** (Modeling Activity #9 in text) Change, Difference Equations & Proportionality Relationships
Day 3, PM:	Looking for Proportionality and Change Patterns in Tables and Graphs **Light Intensity** (Modeling Activity #9 in text)
Day 4, AM	Building Bridges Revisited **Bouncing Ball** (Modeling Activity #5 in text) **Distance & Velocity Graphs**
Day 4, PM	Classifying Mathematical Models Analyzing (Aids) Epidemic Data Models of Population Growth (Exponential, Logistic, Predator/Prey) Models, Interpretations, Representations, & Logic Barnyards & Tournaments (Ranking Schemes) Chicken Pecking Problem Ranking Athletic Teams (Modeling Activity #11 in text)
Day 5, AM:	Newton's Law of Cooling & Stella Model Predator/Prey Simulation & Stella Model
Day 5, PM:	Modeling Process Revisited, Examples, & relation to Logic Open for Discussion of Projects, Portfolios, Issues
Day 6 & 7:	Project Presentations

Our Approach to Modeling from Data

Given a real world phenomenon to consider, we ask our students to demonstrate the following steps in fitting a model to data relative to the phenomenon:

- Formulate the key problem or question. (Show you understand the problem.)
- Communicate your preconceptions.
- Discuss the limitations, assumptions, and scope of your investigation.
- Collect and organize data.
- Analyze and interpret data.
- Choose and fit an appropriate model by varying parameters and testing goodness of fit by an identified numerical criterion (sum of errors, average error, percent error).
- Validate, summarize and report findings. Perhaps you can identify a proportionality relationship to validate your choice of models. (reflect, describe, formulate, evaluate, support, generalize, research, and suggest.)

Sample CBL/CBR Activities

1. A Bouncing Golf Ball

We use a TI 82 calculator and a calculator-based ranger (CBR) to help students develop and evaluate a mathematical model to describe the motion of bouncing golf ball. Students are asked to create a model appropriate for considering questions such as the following:

- Can you find a model to predict the ball's height at any time between two specified successive bounces? (Perhaps between the first and second or between second and third time it hits the floor?)
- How high will the ball bounce on the n^{th} bounce?
- Can we predict the ball's velocity at any time between two specified successive bounces?
- How will an individual ball's characteristics affect our results?

Students communicate their preconceptions by (a) sketching graphs predicting the following relationships, and (b) explaining in words the nature of the anticipated relationships.

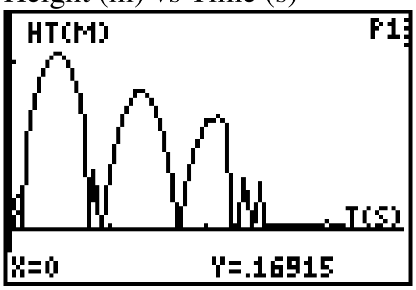
- The height of the ball above the floor as a function of time since it was dropped,
- The height of the ball as a function of the bounce number,
- The ball's velocity as a function of time since it was dropped, and
- The ball's acceleration as a function of time since it was dropped.

We also discuss physical factors that need to be considered as they address the questions for this activity. What factors will they actually consider and what factors will they choose to ignore?

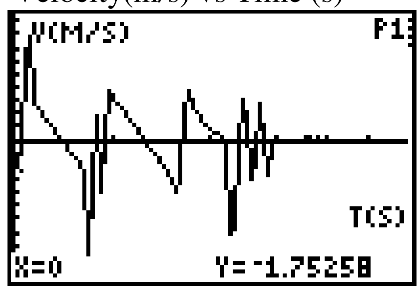
We provide instructions and materials to gather and display the data relevant to this activity (TI 82's and CBR's). For example, see the exploration des cribed on the TI web site with URL **<u>www.ti.com\calc\docs\act\hsmotion08.htm</u>**.

Here are sample height-time and velocity-time graphs from a TI 82 LCD screen.

Height (m) vs Time (s)



Velocity(m/s) vs Time (s)



We use the TI's trace feature and determine the times at which the ball hits the ground the second and third times; we also find the time the ball reaches its maximum height on the second bounce. We also record the ball's height at those three times.

We let t = time elapsed, in seconds, and
h(t) = ball's height, in meters, at time t,

and in this case we decide

(*) $h(0.95) \approx 0.0$, $h(1.33) \approx 0.83$, and $h(1.72) \approx 0.0$.

We can also decide that the ball's maximum heigh ts on three successive bounces are

(**) $h(0.47) \approx 1.05$, $h(1.33) \approx 0.83$, and $h(2.11) \approx 0.67$.

Using qualitative and quantative properties of the Height vs Time graph and model fitting heuristics we have developed previously, our students are likely to conjecture a quadratic relationship exists between the ball's height and time since a bounce started. (We do not allow our students to use the TI's curve fitting routines during model formulation.) So, the students can fit a quadratic relationship to the second bounce using either the points established in (*) above or data stored in their TI 82.

Our students would probably derive an approximate relationship close to

(★) $h(t) \approx -5.6(t - 1.33)^2 + 0.83 \approx -5.6t^2 + 14.9t - 9.08$, for $0.95 \leq t \leq 1.72$

for the second bounce. Note that the coefficient of t^2 is not the expected –4.9. (Although our students probably will not take notice of that fact.) That fact should lead to some interesting discussions during the model validation step. (We have found that our students do not always generate "good" data.)

Using the data in (**) above, we hope students are lead to conjecture that the ball's maximum height on any bounce is approximately 0.8 its maximum height on the previous bounce. If b(n) denotes the maximum height of the n^{th} bounce, then for each bounce after the first, $b(n) = 0.8b(n-1)$. Or, in general $b(n) = (0.8)^{n-1}b(1)$. Of course, our students must gather data on more bounces to validate this model.

Visiting the Velocity vs Time graph we use the TI 82's trace feature to examine the ball's velocity between 0.95 sec and 1.72 sec.

We let t = time elapsed, in seconds, and
v(t) = velocity, in meters per second, of the ball t time t.

About the best we can do in this case is

(***) $v(0.99) \approx 3.73$, $v(1.33) \approx 0.16$, $v(1.38) \approx -0.31$, $v(1.68) \approx -3.99$.

In the case, our students would probably conjecture a linear relationship between the ball's velocity and the time since a bounce started. Our students would derive an approximate relationship close to

(★★) $v(t) \approx -11.2t + 14.82$, for $0.99 \leq t \leq 1.68$

which closely approximates the derivative of the function for h in (★) above.

Note, $h'(t) \approx -11.2t + 14.9$.

Discussion: How does our approach to this activity differ from the approach typically found in published versions?

2. Intensity of a Light Source

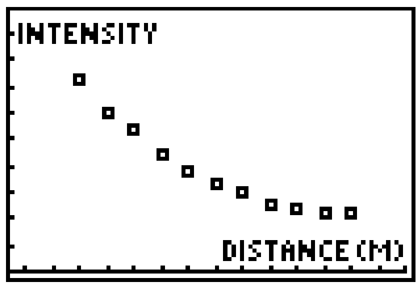
We use a TI 82 calculator, a TI calculator based laboratory (CBL) system unit, and a light sensor to help students develop and evaluate a mathematical model for the relationship between the intensity of a light source, in Watts per square meter, and distance from the source, in meters.

As in the previous activity, students communicate their preconceptions by (a) sketching graphs predicting the nature of the relationship, and (b) explaining in words the nature of the anticipated relationship.

Once again, we also discuss physical factors that need to be considered as they address the question for this activity. What factors will they actually consider and what factors will they choose to ignore?

We provide instructions and materials to gather and display the data relevant to this activity (TI 82's and CBL's and light probes). For example, see the exploration described on pages 54-56 of TI's "CBL System Workbook."

Here is a sample Intensity vs Distance graph from a TI 82 LCD screen.



We have found it useful to transfer experimental data from the TI 82 to an Excel spreadsheet. Doing so allows our students to explore "what if" scenarios.

Here is the data behind the graph in an Excel spreadsheet with some analysis.

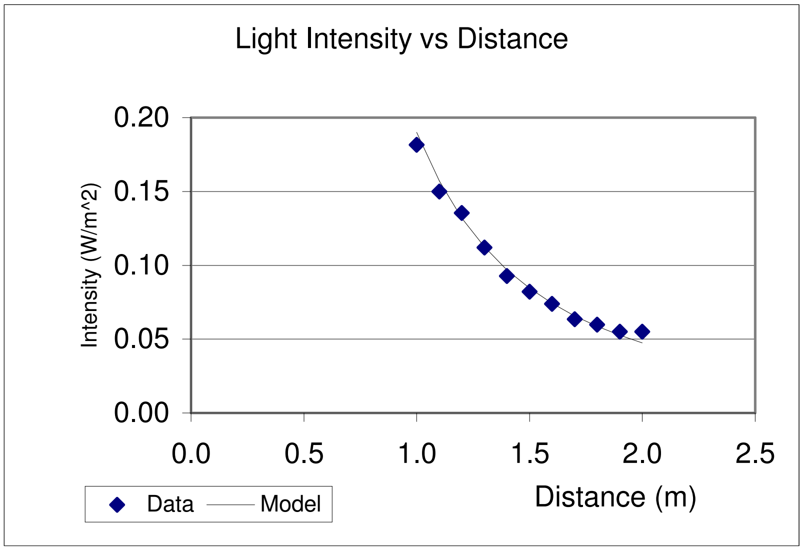
distance (m)	intensity (W/m^2)			
s	**I**	**s*I**	**(s^2)*I**	**Model**
1.0	0.1815	0.1815	0.1815	0.1900
1.1	0.1499	0.1649	0.1814	0.1570
1.2	0.1355	0.1626	0.1951	0.1319
1.3	0.1121	0.1457	0.1894	0.1124
1.4	0.0928	0.1299	0.1819	0.0969
1.5	0.0821	0.1232	0.1847	0.0844
1.6	0.0739	0.1182	0.1892	0.0742
1.7	0.0635	0.0635	0.1835	0.0657
1.8	0.0598	0.0658	0.1938	0.0586
1.9	0.0550	0.0660	0.1986	0.0526
2.0	0.0550	0.0715	0.2200	0.0475

It appears that Intensity %1/(Distance)^2. So, in this case, our students might formulate the following model.

$$I \approx 0.19/s^2,$$

where I denotes the light's intensity and s denotes the distance from the light source.

We compare the model's predictions with the actual data in the graph below.



It looks like the model is a good fit to the data. However, we ask our students to develop, justify, and apply their own numerical criteria for goodness of fit. As noted above, we do not use the calculator's curve fitting procedures during the model building steps.

Discussion: Why do we not allow our students to use the TI 82's or Excel's curve fitting tools in the development of their mathematical models? Why do we ask our students to develop their own "goodness of fit" criterion?

3. Cooling a Thermometer

We use a TI 82 calculator, a CBL unit, and a temperature probe to help students develop a mathematical model for a cooling phenomenon. In this case a temperature probe is placed in a cup of hot water until it records the temperature of the hot water in degrees Celsius, and then it is placed in a cup of room temperature water. Students will develop a model for the relationship between the probes temperature and the time it has been in the room temperature water.

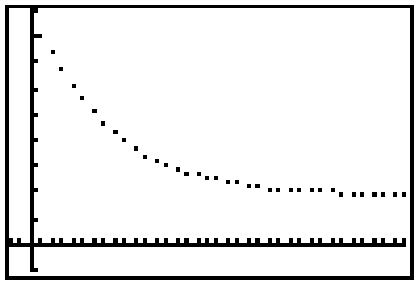
As in the previous activities, students communicate their preconceptions, both graphically and in writing, regarding the relationship being investigated. In this case, we also ask them to comment on the rate at which they predict the probe will cooling.
Will they predict the rate of cooling is constant? We also ask the students to describe how they will approach the following tasks:

- Determine the relationship between the temperature recorded and the length of time the probe has been in room temperature water.
- Examine the rate of cooling.

As before, we discuss relevant factors that might affect our results.

Again we supply instructions and materials to gather and display data relevant to this investigation. For example, see pages 27-31 and 45-47 of TI's "CBL System Workbook."

Here is a sample Temperature vs Time graph from a TI 82 LCD screen.



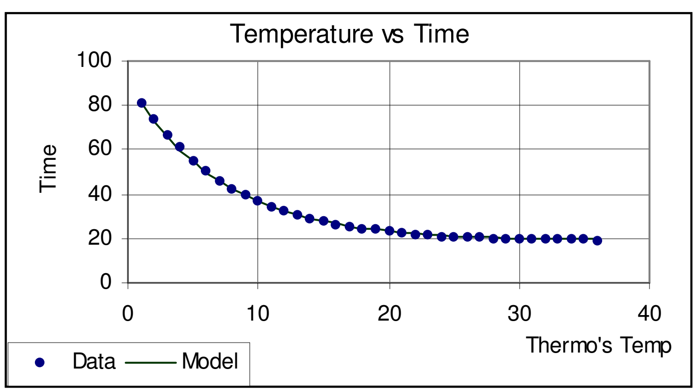
In this case the room temperature was 18.9°C. As with the investigation of light intensity, to facilitate our analysis, we transfer the experimental data from the TI 82 to an Excel spreadsheet that is printed on the following page.

Experimenting with some 'what if' scenarios can lead students to the conjecture that the average rate of change in the temperature of the probe, in degrees/sec, is proportional to the difference between the current probe temperature and the room temperature.

So, if we let P(n) = the probe's temperature after n seconds, and we assume the room temperature is 18.9, then our relationship can be modeled by the difference equation

$$P(1) = 81.1$$
$$P(n) - P(n-1) = k[P(n-1) - 18.9]$$

The graph below shows that when k = -0.13 the model above is a good fit to the data.



Here is a copy of the spreadsheet.

Time (sec) t	Temperature (Celsius) C	Rate of Change in Temperature (degrees/sec)	Difference in Probe Temp and Water Temp	Ratio of Rate of Change to Difference in Probe Temp and Water Temp
1	81.1		62.2	
2	74.3	-6.8	55.4	-0.123
3	67.1	-7.2	48.2	-0.149
4	60.9	-6.2	42.0	-0.148
5	55.4	-5.5	36.5	-0.151
6	50.5	-4.9	31.6	-0.155
7	46.3	-4.3	27.4	-0.155
8	42.5	-3.8	23.6	-0.159
9	39.2	-3.3	20.3	-0.163
10	36.5	-2.7	17.6	-0.153
11	34.1	-2.4	15.2	-0.158
12	32.0	-2.1	13.1	-0.160
13	30.2	-1.8	11.3	-0.159
14	28.7	-1.5	9.8	-0.153
15	27.5	-1.2	8.6	-0.140
16	26.4	-1.1	7.5	-0.147
17	25.5	-0.9	6.6	-0.136
18	24.7	-0.8	5.8	-0.138
19	23.9	-0.8	5.0	-0.160
20	23.1	-0.8	4.2	-0.190
21	22.4	-0.7	3.5	-0.200
22	21.9	-0.5	3.0	-0.167
23	21.5	-0.4	2.6	-0.154
24	21.1	-0.4	2.2	-0.182
25	20.8	-0.3	1.9	-0.158
26	20.6	-0.2	1.7	-0.118
27	20.3	-0.3	1.4	-0.214
28	20.0	-0.3	1.1	-0.273
29	20.0	0.0	1.1	0.000
30	19.8	-0.2	0.9	-0.222
31	19.7	-0.1	0.8	-0.125
32	19.6	-0.1	0.7	-0.143
33	19.5	-0.1	0.6	-0.167
34	19.4	-0.1	0.5	-0.200
35	19.4	0.0	0.5	0.000
36	19.3	-0.1	0.4	-0.250

Students can also develop an exponential function to model this phenomenon. In this case the probe's temperature after t seconds, P(t), can be approximated by

$$P(t) \approx 62.2\, e^{-0.13(t-1)} + 18.9.$$

Discussion: How might we have used the concept of percent change to motivate the development of a model for the cooling phenomenon?

Additional Information

Web Site for the mathematical modeling workshop for middle school teachers:

http://faculty.salisbury.edu/~dccathca/ModelsWorkshop/description.html

Web sites for two other mathematical modeling courses:

A course for prospective elementary school teachers-

http://faculty.salisbury.edu/~dccathca/MATH115/abstract.htm

A course for upper level mathematics majors-

http://faculty.salisbury.edu/~dccathca/MATH465/Syllabus.htm