If A = $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 2 & -4 & 4 \end{bmatrix}$, describe *Col* A and *Nul* A.

Col A = $\{\bar{b} \in R^3 : A\bar{x} = \bar{b} \text{ for some } \bar{x} \in R^4\}$

That is $\bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is in *Col* A iff A$\bar{x}$ = $\bar{b}$ is consistent.

Consider $\begin{bmatrix} 1 & -1 & 0 & 0 & b_1 \\ 0 & 1 & -2 & 2 & b_2 \\ 0 & 2 & -4 & 4 & b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0-2 & 2 & b_1 + b_2 \\ 0 & 1 & -2 & 2 & b_2 \\ 0 & 0 & 0 & 0 & -2b_2 + b_3 \end{bmatrix}$.

So, $\bar{b}$ is in *Col* A iff $0b_1 - 2b_2 + b_3 = 0$.

Hence *Col* A consists of vectors of the form $\begin{bmatrix} b_1 \\ \frac{1}{2}b_3 \\ b_3 \end{bmatrix}$ where b_1 and b_3 are free.

Alternatively, *Col* A consists of vectors of the form $s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ \frac{1}{2} \\ 1 \end{bmatrix}$ for s, t scalars.

Now, *Nul* A = $\{\bar{x} \in R^4 : A\bar{x} = \bar{0}\}$

Consider $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 2 & 0 \\ 0 & 2 & -4 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0-2 & 2 & 0 \\ 0 & 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

So $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in$ *Nul* A iff $x_1 = 2x_3 - 2x_4$ and $x_2 = 2x_3 - 2x_4$ where x_3 and x_4 are free.

Alternatively, *Nul* A consists of vectors of the form $s\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$ for s, t scalars