*Some Important Results in Chapter 1*

**Theorem 2. Existence and Uniqueness Theorem**

A linear system is consistent iff the rightmost column of the augmented matrix is not a pivot column. The solution set for a consistent system is unique iff there are no free variables.

**Theorem 3.**

If A is an $m \times n$ matrix with columns $\mathbf{a}_1, \ldots, \mathbf{a}_n$, and if $\mathbf{b}$ is in $R^m$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has the same solution set as the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots + x_n\mathbf{a}_n$ which has the same solution set as the linear system whose augmented matrix is $[\mathbf{a}_1\ \ \mathbf{a}_2\ \ \ldots\ \ \mathbf{a}_n\ \ \mathbf{b}]$.

**Theorem 4.**

Let A be an $m \times n$ matrix. The following statements are logically equivalent.

a. For each $\mathbf{b}$ in $R^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
b. Each $\mathbf{b}$ in $R^m$ is a linear combination of the columns of A.
c. The columns of A span $R^m$.
d. A has a pivot in every row.

**Theorem 7. Characterization of Linearly Dependent Sets**

An indexed set of vectors $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of two or more vectors is linearly dependent iff at least one of the vectors in $S$ is a linear combination of the others.

**Theorem 8.**

If a set contains more vectors then there are entries in each vector, then the set is linearly dependent.

**Theorem 9.**

If a set contains the zero vector, then the set is linearly dependent.

**Theorem 10.**

Let $T: R^n \to R^m$ be a linear transformation. The there exists a unique m x n matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $R^n$. In fact A is the matrix whose $j^{th}$ column is the vector $T(\mathbf{e}_j)$.

**Theorem 11.**

Let $T: R^n \to R^m$ be a linear transformation. $T$ is 1-1 iff $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.

**Theorem 12.** Let $T: R^n \to R^m$ be a linear transformation with standard matrix A. Then:

a. $T$ maps $R^n$ onto $R^m$ iff the columns of A span $R^m$.
b. $T$ is 1-1 iff the columns of A are linearly independent.