$$\begin{cases} 3x + y - z = 0 \\ 2x + y = 0 \\ x - y + z = 0 \end{cases} \qquad E_1 \leftrightarrow E_3 \begin{cases} x - y + z = 0 \\ 2x + y = 0 \\ 3x + y - z = 0 \end{cases}$$

$$-2E_1 + E_2 \to E_2 \begin{cases} x - y + z = 0 \\ 3y - 2z = 0 \\ 3x + y - z = 0 \end{cases}$$

$$-3E_1 + E_3 \to E_3 \begin{cases} x - y + z = 0 \\ 3y - 2z = 0 \\ 4y - 4z = 0 \end{cases}$$

$$\tfrac{1}{4}E_3 \to E_3 \begin{cases} x - y + z = 0 \\ 3y - 2z = 0 \\ y - z = 0 \end{cases}$$

$$E_2 \leftrightarrow E_3 \begin{cases} x - y + z = 0 \\ y - z = 0 \\ 3y - 2z = 0 \end{cases}$$

$$1E_2 + E_1 \to E_1 \begin{cases} x = 0 \\ y - z = 0 \\ 3y - 2z = 0 \end{cases}$$

$$-3E_2 + E_3 \to E_3 \begin{cases} x = 0 \\ y - z = 0 \\ z = 0 \end{cases}$$

$$1E_3 + E_2 \to E_2 \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

So, our unique solution is $(0, 0, 0)$

Solve the linear system

$$\begin{cases} x + y + z = 0 \\ \quad\;\; y + 2z = 0 \end{cases}$$

$$-1E_2 + E_1 \to E_1 \quad \begin{cases} x \qquad -z = 0 \\ \quad\; y + 2z = 0 \end{cases}$$

In this case we have infinitely many solutions that we can generate by assigning a value to z and determining what ~~value~~ values must be assigned to x and y. So, we choose (x, y, z) so that

$$x = z \quad , \text{and.}$$
$$y = -2z$$

where z can be any number.

We write

$$\begin{cases} x = z \\ y = -2z \\ z \text{ is free} \end{cases}$$

We conclude that there are infinitely many solutions of the form $(t, -2t, t)$ where t is any real number.

Solve:

$$\begin{cases} x + y = 2 \\ 2x + 3y + z = 4 \\ x + 2y + 2z = 6 \end{cases}$$

$-2E_1 + E_2 \to E_2$ $$\begin{cases} x + y = 2 \\ y + z = 0 \\ x + 2y + 2z = 6 \end{cases}$$

$-1E_1 + E_3 \to E_3$ $$\begin{cases} x + y = 2 \\ y + z = 0 \\ y + 2z = 4 \end{cases}$$

$-1E_2 + E_3 \to E_3$ $$\begin{cases} x + y = 2 \\ y + z = 0 \\ z = 4 \end{cases}$$

At this point we can see that our unique solution is $(6, -4, 4)$.

$-1E_2 + E_1 \to E_1$ $$\begin{cases} x - z = 2 \\ y + z = 0 \\ z = 4 \end{cases}$$

$1E_3 + E_1 \to E_1$

$-1E_3 + E_2 \to E_2$ $$\begin{cases} x = 6 \\ y = -4 \\ z = 4 \end{cases}$$ (Row Reduced Form)

Of course we can see here also that our unique solution is $(6, -4, 4)$