The Invertible Matrix Theorem

Let A be an n x n matrix. Then the following statements are equivalent.

a. A is an invertible matrix.

b. A is row equivalent to I_n.

c. A has n pivot positions

d. A$\mathbf{x} = \mathbf{0}$ has only the trivial solution.

e. The columns of A form a linearly independent set.

f. The linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is 1-1.

g. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in R^n$.

h. The columns of A span R^n .

i. The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto R^n.

j. There is an $n \times n$ matrix C such that $CA = I_n$.

k. There is an $n \times n$ matrix D such that $AD = I_n$.

l. A^T is invertible.

m. The columns of A form a basis of R^n.

n. Col A = R^n

o. Dim Col A = n

p. Rank A = n

q. Nul A = $\{\mathbf{0}\}$

r. Dim Nul A = 0

s. Det A $\neq$ 0