

## Linear Independence

Suppose  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ . Find  $x_1$ ,  $x_2$ , and  $x_3$  such that  $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$ , where  $\mathbf{0}$  is the zero vector in  $\mathbf{R}^3$ .

An indexed set of vectors  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_n\}$  in  $\mathbf{R}^n$  is said to be *linearly independent* iff the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + \dots + x_n\mathbf{v}_n = \mathbf{0}$$

has only the trivial solution. Otherwise, the set of vectors is said to be *linearly dependent*.

Suppose  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix}$ . Determine whether or not the set  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is linearly independent.

Suppose  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 2 & 2 & 4 \end{bmatrix}$ . Are the columns of  $A$  linearly independent?

Suppose  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ,  $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ , and  $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ .

Is the set  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  linearly independent?

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Do the columns of  $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 5 & 8 \end{bmatrix}$  span  $\mathbb{R}^2$ ? Are the columns of  $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 5 & 8 \end{bmatrix}$  linearly independent?

Are the columns of  $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 0 & 3 \end{bmatrix}$  linearly independent?