Matrices and Vectors

An ***m x n matrix*** is a rectangular array of numbers with m rows and n columns. In the matrix below a_{ij} denotes the entry in the i^{th} row and j^{th} column.

$$A_{mxn} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]_{mxn}$$

Vectors in $\mathbf{R}^n$

If n is a positive integer, $\mathbf{R}^n$ is the set of all ordered n-tuples of real numbers. We will usually write the elements of R^n as n x 1 column matrices, such as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix}$$

We call the elements of $\mathbf{R}^n$ ***vectors***. A vector whose entries are all zero is called the ***zero vector*** and is denoted by $\mathbf{0}$.

Suppose

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_n \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ w_n \end{bmatrix} \in \mathbf{R}^n \text{ and } c \in \mathbf{R}.$$

We say vectors **u** and **v** are ***equal*** iff $u_i = v_i$ for all $i = 1, 2, \ldots, n$. In this case we write $\mathbf{u} = \mathbf{v}$.

The ***scalar multiple*** of **u** by c, denoted by c**u**, is defined by

$$c\mathbf{u} = \begin{bmatrix} cu_1 \\ cu_2 \\ . \\ . \\ cu_n \end{bmatrix}$$

The ***sum*** of **u** and **v**, denoted by **u** + **v**, is defined by $\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ . \\ . \\ u_n + v_n \end{bmatrix}$.

Suppose $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p} \in \mathbf{R}^n$ and $c_1, c_2, \ldots, c_p \in \mathbf{R}$, a vector defined by

$$\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v_2} + \ldots + c_p\mathbf{v_p}$$

is called a ***linear combination*** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ with weights $c_1, c_2, \ldots, c_p$.

Geometric Interpretations (See pp. 29-31.)

Algebraic properties of vectors in $\mathbf{R}^n$. (See p. 32.)

Vector equations, linear systems, and augmented matrices.

Suppose $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p} \in \mathbf{R}^n$, then the set of all linear combinations of $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$ is called the *subset of* $\boldsymbol{R}^n$ *spanned* (or *generated*) *by* $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$. we denote this subset by Span$\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}\}$.

Exercises:

a. Solve:

$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 11 \\ x_3 &= 3 \\ 2x_1 + 4x_2 + 5x_3 &= 25 \end{aligned}$$

b. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 11 \\ 3 \\ 25 \end{bmatrix}$.

Find c_1, c_2, and c_2 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$.
Is **b** in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

c. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 11 \\ 3 \\ 28 \end{bmatrix}$.

Find c_1, c_2, and c_2 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{c}$.

d. Is **c** in Span{$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$}?

e. Give a geometric description of Span{$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$}?

f. How many vectors are in Span{$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$}?