

## Section 1.2: Row Reduction and Echelon Forms

**Echelon form (or row echelon form):**

1. All nonzero rows are above any rows of all zeros.
2. Each *leading entry* (i.e. left most nonzero entry) of a row is in a column to the right of the leading entry of the row above it.
3. All entries in a column below a leading entry are zero.

**EXAMPLE:** Echelon forms

$$\begin{array}{c}
 \text{(a)} \left[ \begin{array}{cccccc}
 \blacksquare & * & * & * & * \\
 0 & \blacksquare & * & * & * \\
 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0
 \end{array} \right]
 \end{array}
 \qquad
 \begin{array}{c}
 \text{(b)} \left[ \begin{array}{ccc}
 \blacksquare & * & * \\
 0 & \blacksquare & * \\
 0 & 0 & \blacksquare \\
 0 & 0 & 0
 \end{array} \right]
 \end{array}$$
  

$$\text{(c)} \left[ \begin{array}{cccccccccccc}
 0 & \blacksquare & * & * & * & * & * & * & * & * & * \\
 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * & * \\
 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * & * & * \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * & *
 \end{array} \right]$$

**Reduced echelon form:** Add the following conditions to conditions 1, 2, and 3 above:

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

**EXAMPLE (continued):**

Reduced echelon form :

$$\left[ \begin{array}{cccccccccccc}
 0 & 1 & * & 0 & 0 & * & * & 0 & 0 & * & * \\
 0 & 0 & 0 & 1 & 0 & * & * & 0 & 0 & * & * \\
 0 & 0 & 0 & 0 & 1 & * & * & 0 & 0 & * & * \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & * & * \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * & *
 \end{array} \right]$$

**Theorem 1 (Uniqueness of The Reduced Echelon Form):**

Each matrix is row-equivalent to one and only one reduced echelon matrix.