Theorem 4 Let ***A*** be an *m x n* matrix. Then the following statements are logically equivalent.

a. For each $\mathbf{b} \in \mathbf{R}^n$, the equation $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ has a solution.
b. Each $\mathbf{b} \in R^n$ is a linear combination of the columns of ***A***.
c. The columns of ***A*** span R^n.
d. ***A*** has a pivot position in every row.

Some Algebraic Properties of R^n

For all **u**, **v**, **w** $\in R^n$ and all scalars *c* and *d*:

(i) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
(ii) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
(iii) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
(iv) $\mathbf{u} + (-\mathbf{u}) = -\mathbf{u} + \mathbf{u} = \mathbf{0}$ where $-\mathbf{u}$ denotes $-1\mathbf{u}$
(v) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
(vi) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
(vii) $c(d\mathbf{u}) = (cd)\mathbf{u}$
(viii) $1\mathbf{u} = \mathbf{u}$

Some Algebraic Properties of the Matrix-Vector Product

For all *m x n* matrices A, all **u**, **v** $\in R^n$, and all scalars *c*:

(i) $A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
(ii) $A(c\mathbf{u}) = c(A\mathbf{u})$