Linear Algebra -

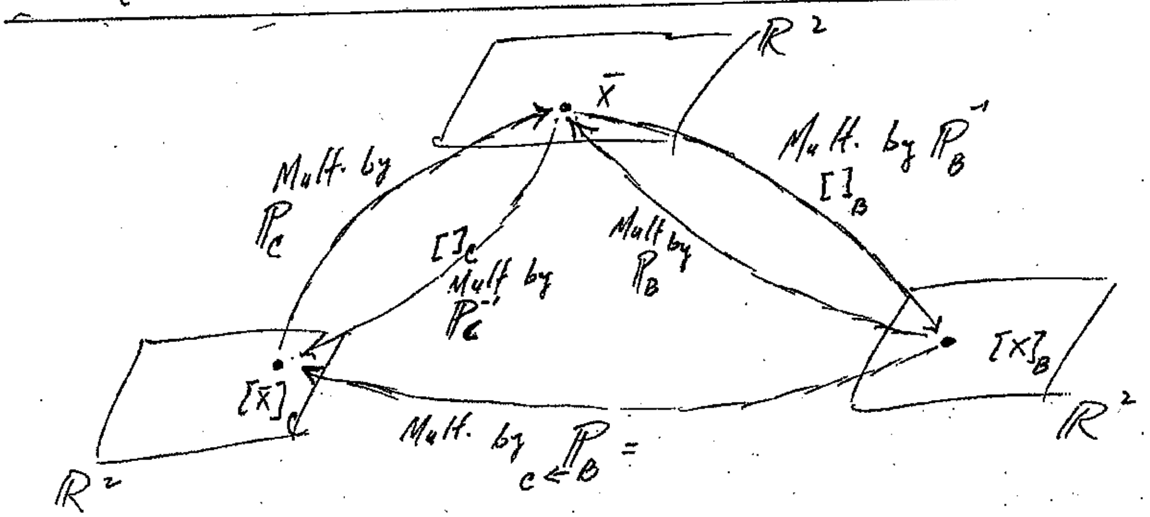
Recall our work from session #33. We considered $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix} \right\}$ bases for $\mathbb{R}^2$. Associated with those bases are two coordinate mappings $\bar{x} \mapsto [\bar{x}]_{\mathcal{B}}$ and $\bar{x} \mapsto [\bar{x}]_{\mathcal{C}}$. Those mappings are 1-1 linear transformations from $\mathbb{R}^2$ onto $\mathbb{R}^2$. Consequently, their inverse mappings $[\bar{x}]_{\mathcal{B}} \mapsto \bar{x}$ and $[\bar{x}]_{\mathcal{C}} \mapsto \bar{x}$ are also 1-1 linear transformations. Associated with those coordinate mappings are the following matrices:

$P_{\mathcal{B}} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ is the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^2$

$P_{\mathcal{B}}^{-1} = [\quad]$

$P_{\mathcal{C}} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

$P_{\mathcal{C}}^{-1} = [\quad]$



Theorem: Let $\mathcal{B} = \{\bar{b}_1, \ldots, \bar{b}_n\}$ and $\mathcal{C} = \{\bar{c}_1, \ldots, \bar{c}_n\}$ be bases for a vector space V. Then there is a unique $n \times n$ matrix $\underset{C \leftarrow B}{\mathbb{P}}$ such that

$$[\bar{x}]_C = \underset{C \leftarrow B}{\mathbb{P}} [\bar{x}]_B .$$

That matrix is called the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

Remark: $\underset{C \leftarrow B}{\mathbb{P}} = \mathbb{P}_C^{-1} \mathbb{P}_B$

Example: Let $\bar{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, $\bar{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$, $\bar{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\bar{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and consider the bases $\mathcal{B} = \{\bar{b}_1, \bar{b}_2\}$, $\mathcal{C} = \{\bar{c}_1, \bar{c}_2\}$ of $\mathbb{R}^2$.

Let's find $\underset{C \leftarrow B}{\mathbb{P}}$.