Coordinate Vectors

Suppose $B = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p\}$ is a basis for a subspace H. For each $\mathbf{x} \in$ H, the *coordinates of* $\boldsymbol{x}$ *relative to B* are the weights $c_1, c_2, \ldots, c_p$ such that

$$\mathbf{x} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \ldots + c_p\mathbf{b}_p \,,$$

and the vector in R^p

$$[\mathbf{x}]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}$$

Is called the *coordinate vector of* $\boldsymbol{x}$ *relative to B* or the *B-coordinate vector of* $\boldsymbol{x}$**.**

Suppose A = $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 2 & -4 & 4 \end{bmatrix}$.

We have seen that a basis for *Col* A is $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$.

Is $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$ in *Col* A? If so, find $[\mathbf{x}]_B$, the *B*-coordinate vector of $\mathbf{x}$.