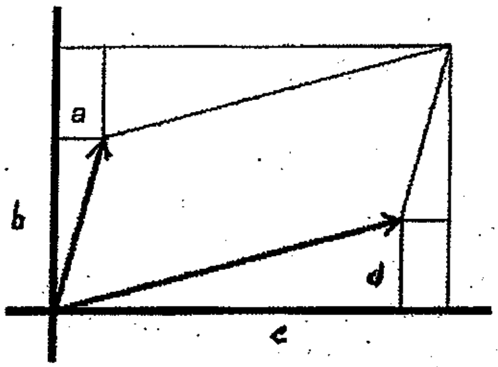
# Linear Algebra

(1)




Find the area of the parallelogram determined by the vectors $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} c \\ d \end{bmatrix}$

(2) What must be the case regarding the values of $a$, $b$, $c$, and $d$ for the matrix $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ to have an inverse?

## Determinant Function on Square Matrices

For any matrix $A$, the matrix obtained by deleting the $i^{th}$ row and $j^{th}$ column of $A$ is called the $ij^{th}$ minor of $A$ and is denoted by $A_{ij}$. For any $n \times n$ matrix $A$, the determinant of $A$, denoted by $\det A$ is defined by $\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$. If $A$ is $1 \times 1$ where $A = [a]$, $\det A = a$.

③ Suppose $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$. Calculate $\det A$.

④ Suppose $\mathbb{B} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 4 \\ 2 & 2 & 5 \end{bmatrix}$. Calculate $\det \mathbb{B}$.

# General Definition of Determinant

Suppose $A = [a_{ij}]$ is $n \times n$.
We denote by $S_n$ the set of all permutations of the set $\{1, 2, \dots, n\}$.

Definition If $A$ is an $n \times n$ matrix, the determinant of $A$, denoted by $\det A$ or $|A|$ is defined as follows:

$$|A| = \sum_{\alpha \in S_n} (\operatorname{sign} \alpha)\, a_{1\alpha}\, a_{2\alpha} \cdots a_{n\alpha}.$$

## Properties of Determinants

Suppose $A$ is $n \times n$

1) If a multiple of one row of $A$ is added to another row of $A$ to produce a matrix $B$, then $|A| = |B|$.
2) If two rows of $A$ are interchanged to produce $B$, then $|B| = -|A|$
3) If one row of $A$ is multiplied by $k$ to produce $B$, then $|B| = k|A|$.
4) $A$ is invertible iff $|A| \neq 0$
5) $|A| = |A^T|$
6) If $B$ is $n \times n$, $|AB| = |A| \cdot |B|$

A $m \times n$ upper triangular matrix is one whose entries below the main diagonal are 0's. How may we calculate the determinant of a square upper triangular matrix?

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

$\det A = |A| =$

Compute $\det A$, where $A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$

$\det A = \begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -4 & 2 \\ 0 & 0 & -5 \\ 0 & 3 & 2 \end{vmatrix} =$