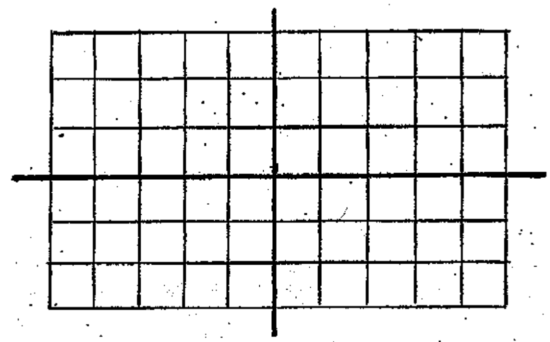
Linear Algebra

Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$, $\bar{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Graph the vectors $\bar{u}$, $A\bar{u}$, $\bar{v}$, $A\bar{v}$.



An <u>eigenvector</u> of an $n \times n$ matrix A is a nonzero vector $\bar{x}$ such that $A\bar{x} = \lambda \bar{x}$ for some scalar λ. A scalar λ is called an <u>eigenvalue</u> of A if there is a nontrivial solution $\bar{x}$ of $A\bar{x} = \lambda \bar{x}$; such an $\bar{x}$ is called an eigenvector corresponding to λ.

In the example above, we can see that 2 is an eigenvalue of $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to 2. Describe the set of all eigenvectors of A corresponding to 2.

In order for any scalar λ to be an eigenvalue for $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$, the equation $A\bar{x} = \lambda \bar{x}$ must have a nontrivial solution. Note,

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ iff } \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{iff } \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{iff } \left(\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{iff } \begin{bmatrix} 3-\lambda & -2 \\ 1 & -\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, a nontrivial solution exists provided $\det \begin{bmatrix} 3-\lambda & -2 \\ 1 & -\lambda \end{bmatrix} = 0$. That is, provided

$$(3-\lambda)(-\lambda) - (1)(-2) = 0.$$

So, in this example, $A\bar{x} = \lambda\bar{x}$ will have a nontrivial solution provided

$$\boxed{\lambda^2 - 3\lambda + 2 = 0}$$

In this example A has eigenvalues 2 and 1.

Let's find the eigenvectors corresponding to 1.

Now let's find the eigenvalues and corresponding eigenvectors for

$$\mathbb{B} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}.$$