Linear Algebra 11/29/06

Determine the eigenvalues and corresponding eigenvectors for the matrix $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$. Also, find a basis for the eigenspace corresponding to each eigenvalue.

We seek $\lambda$ so that $\begin{vmatrix} (4-\lambda) & 2 & 3 \\ -1 & (1-\lambda) & -3 \\ 2 & 4 & (9-\lambda) \end{vmatrix} = 0.$

$$\begin{vmatrix} (4-\lambda) & 2 & 3 \\ -1 & (1-\lambda) & -3 \\ 2 & 4 & (9-\lambda) \end{vmatrix} = (4-\lambda)\begin{vmatrix} (1-\lambda) & -3 \\ 4 & (9-\lambda) \end{vmatrix} - (-1)\begin{vmatrix} 2 & 3 \\ 4 & (9-\lambda) \end{vmatrix}$$

$$+ 2\begin{vmatrix} 2 & 3 \\ (1-\lambda) & -3 \end{vmatrix}$$

$$= (4-\lambda)\left[(1-\lambda)(9-\lambda) - (-12)\right] + \left[2(9-\lambda) - 12\right]$$

$$+ 2\left[-6 - (1-\lambda)(3)\right]$$

$$= -\lambda^3 + 14\lambda^2 - 57\lambda + 72$$

$$= -(\lambda-3)(\lambda-3)(\lambda-8)$$

The characteristic equation $-\lambda^3 + 14\lambda^2 - 57\lambda + 72 = 0$ has solutions 8 and 3 where 3 is a solution of algebraic multiplicity two. Hence 2 and 3 are eigenvalues for the matrix **A**.

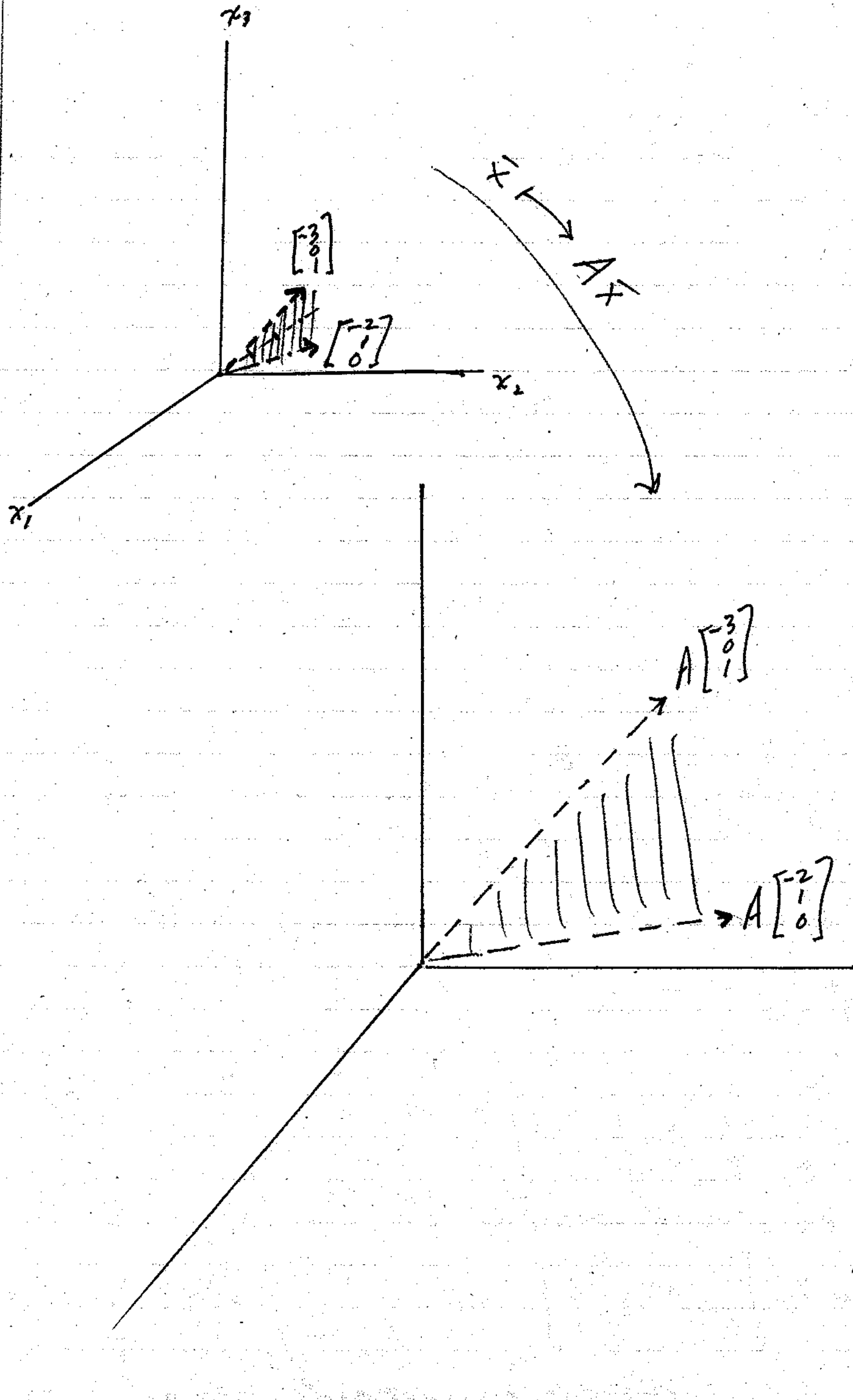
To find eigenvectors corresponding to the eigenvalue 3, we row reduce the augmented matrix for $(A - 3I)\bar{x} = \bar{0}$.

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & -2 & -3 & 0 \\ 2 & 4 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Leftrightarrow x_1 + 2x_2 + 3x_3 = 0$$

So, the general solution for $(A - 3I)\bar{x} = \bar{0}$ is

$\bar{x} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ and $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis

for the eigenspace corresponding to 3.



$x_3$
$x_2$
$x_1$
$\begin{bmatrix}-3\\0\\1\end{bmatrix}$
$\begin{bmatrix}-2\\1\\0\end{bmatrix}$
$\vec{x} \mapsto A\vec{x}$
$A\begin{bmatrix}-3\\0\\1\end{bmatrix}$
$A\begin{bmatrix}-2\\1\\0\end{bmatrix}$