Diagonalize $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$

① Find eigenvalues of A

$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$

characteristic polynomial

$\det(A - \lambda I) = 0$

$\det\left(\begin{bmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix}\right) = 0$

$(6-\lambda)(3-\lambda) + 2 = 0$

$18 - 9\lambda + \lambda^2 + 2 = 0$

$\lambda^2 - 9\lambda + 20 = 0$

$(\lambda - 4)(\lambda - 5) = 0$

$\lambda = 4$ or $\lambda = 5$

② Find eigenvectors associated with $\lambda = 4$ and $\lambda = 5$ respectively

$\lambda = 4$

$\begin{bmatrix} 2 & -1 & 0 \\ 2 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \bar{x} = t\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} = s\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda = 5$

$\begin{bmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \bar{x} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

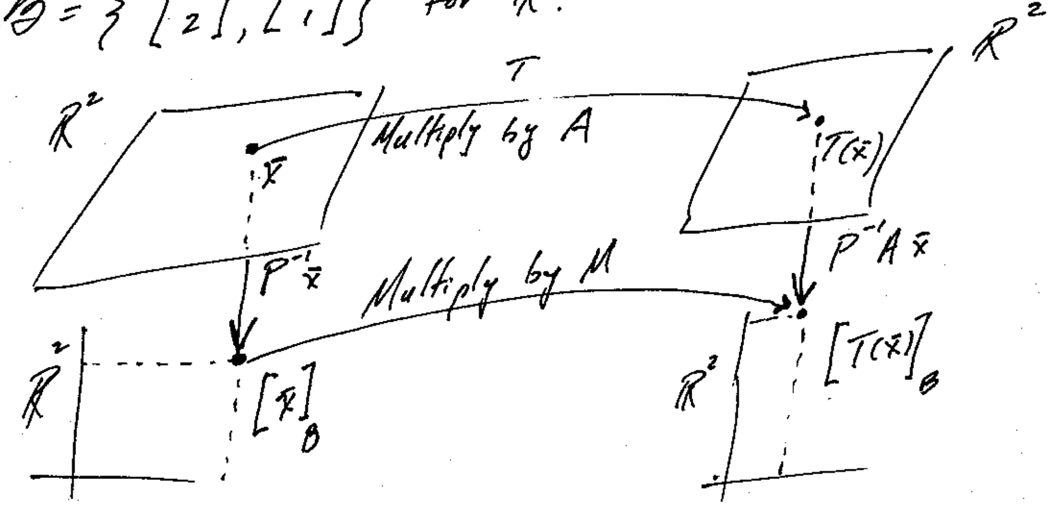
③ Construct P $\quad P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

④ Construct D $\quad D = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$

$A = PDP^{-1}$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

Consider $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\bar{x} \mapsto A\bar{x}$ where $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ and choose the particular basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$. Let $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$.



By previous work we have this special case

$$[T(\bar{x})]_B = M [\bar{x}]_B \quad \text{where}$$

$$M = \left[\left[T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)\right]_B \; \vdots \; \left[T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)\right]_B \right]$$

$$= \left[\begin{bmatrix} 4 \\ 8 \end{bmatrix}_B \; \begin{bmatrix} 5 \\ 5 \end{bmatrix}_B \right]$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$

In this case M is the $\mathcal{B}$-matrix for T and we write $[T]_B = M$.

$$[T(\bar{x})]_B = [T]_B [\bar{x}]_B$$