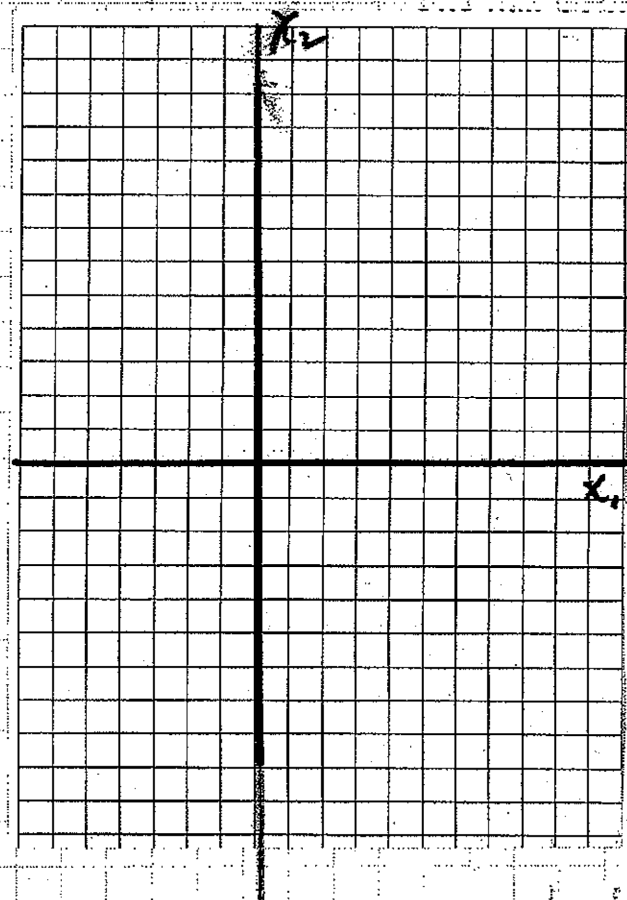
Session #5 Warm Up

1. Solve and Graph the Solution Sets

a) $-2x_1 + x_2 = 0$

b) $-2x_1 + x_2 = 3$



2. Solve and Describe the Solution Sets

a. $\begin{cases} x_1 - 2x_2 + 2x_3 = 0 \\ x_3 = 0 \end{cases}$

b. $\begin{cases} x_1 - 2x_2 + 2x_3 = 3 \\ x_3 = 3 \end{cases}$

Session # 5

Consider the matrix $\mathbb{A} = \begin{bmatrix} 2 & -1 & 4 \\ 1 & -\frac{1}{2} & 3 \\ 4 & -2 & 10 \end{bmatrix}$ and the vector $\bar{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Find those vectors $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $\mathbb{A}\bar{x} = \bar{0}$. Provide a geometric interpretation for the solution set for the equation $\mathbb{A}\bar{x} = \bar{0}$

Linear systems that can be written in the form $\mathbb{A}\bar{x} = \bar{0}$ are called <u>homogeneous systems</u>. Such systems must have at least one solution since $\mathbb{A}\bar{0} = \bar{0}$. That zero solution is called the <u>trivial solution</u>. We are interested in <u>nontrivial solutions</u>.

Find all solutions for $\mathbb{A}\bar{x} = \bar{b}$ where $\mathbb{A} = \begin{bmatrix} 2 & -1 & 4 \\ 1 & -\frac{1}{2} & 3 \\ 4 & -2 & 10 \end{bmatrix}$ and $\bar{b} = \begin{bmatrix} 6 \\ 3 \\ 12 \end{bmatrix}$. Provide a geometric description of that solution set.

1. Solve: $\begin{cases} x_1 - 2x_2 + 2x_3 = 3 \\ \qquad\qquad\quad x_3 = 3 \end{cases}$

2. Solve: $x_1\begin{bmatrix}1\\0\end{bmatrix} + x_2\begin{bmatrix}-2\\0\end{bmatrix} + x_3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix}$

3. Solve: $\begin{bmatrix}1 & -2 & 2\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix}$

4. Can $\begin{bmatrix}3\\3\end{bmatrix}$ be written as a linear combination of $\begin{bmatrix}1\\0\end{bmatrix}$, $\begin{bmatrix}-2\\0\end{bmatrix}$, and $\begin{bmatrix}2\\1\end{bmatrix}$?

5. Is $\begin{bmatrix}3\\3\end{bmatrix}$ in span $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}-2\\0\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$?

Show, by a geometric construction, that the vector $\bar{b}$ can be written as a l.c. of $\bar{v}_1$, $\bar{v}_2$, and $\bar{v}_3$.
Is that representation unique?