The Invertible Matrix Theorem

1. Classify each statement as true or false. In each case A refers to an arbitrary *n x n* matrix and **b** is *n x 1*.

   a. If A**x** = **0,** has only the trivial solution, then A is row equivalent to I_n.

   b. If the columns of A span R^n, then the columns are linearly independent.

   c. The equation A**x** = **b** has at least one solution for each $\mathbf{b} \in R^n$.

   d. If the equation A**x** = **0** has a nontrivial solution, then A has fewer than n pivot positions.

   e. If A^T is not invertible, then A is not invertible.

   f. If the equation A**x** = **b** is inconsistent for some $\mathbf{b} \in R^n$, then A**x** = **0** has nontrivial solutions.

   g. If the equation A**x** = **0** has only the trivial solution, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is 1-1.

   h. The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto R^n if there is an *n x n* matrix C such that CA = I.

2. Suppose T: $R^2 \rightarrow R^2$ is defined by $T(x_1,x_2) = (-5x_1 + 9x_2 , 4x_1 - 7x_2)$. Show that T is invertible and find a formula for T^{-1}.

3. Let T: $R^n \rightarrow R^n$ be an invertible linear transformation, and let S at U be functions from R^n into R^n such that $S(T(\mathbf{x})) = \mathbf{x}$ and $U(T(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x} \in R^n$. Show that $U(\mathbf{v}) = S(\mathbf{v})$ for all $\mathbf{v} \in R^n$.

4. A *subspace* of R^n is any set H in R^n that has three properties:
   a. The zero vector is in H.
   b. For each **u** and **v** in H, the sum **u** + **v** is in H.
   c. For each **u** in H and each scalar c, the vector c**u** is in H.

Suppose $K = \{\mathbf{v} \in R^2 : \mathbf{v} = k\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ for some scalar $k\}$. Is K a subspace of R^2?

Suppose $J = \{\mathbf{v} \in R^2 : \mathbf{v} = \begin{bmatrix} k \\ 4 \end{bmatrix}$ for some scalar $k\}$. Is J a subspace of R^2?