A transformation (function or mapping) is ***linear*** provided:
(i) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all **u**, **v** in the domain of T;
(ii) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all **u** in the domain of T and all scalars c.

Suppose A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, **x =** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is in R^2, and T: $R^2 \rightarrow R^2$ is defined by T(**x**) = A**x**, so that

$$T(x) = A\mathbf{x} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_2 \end{bmatrix}.$$

Is the transformation T defined above a linear transformation?

Suppose **u =** $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and **v** = $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ are in R^2 and c is a scalar.

(i) $T(\mathbf{u} + \mathbf{v}) = T(\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix} = \begin{bmatrix} 1(u_1 + v_1) + 2(u_2 + v_2) \\ 1(u_2 + v_2) \end{bmatrix}$

$= \begin{bmatrix} (1u_1 + 2u_2) + (1v_1 + 2v_2) \\ 1u_2 + 1v_2 \end{bmatrix} = \begin{bmatrix} 1u_1 + 2u_2 \\ 1u_2 \end{bmatrix} + \begin{bmatrix} 1v_1 + 2v_2 \\ 1v_2 \end{bmatrix} = T(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}) + T(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix})$

$= T(\mathbf{u}) + T(\mathbf{v})$

Hence, $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$

(ii) $T(c\mathbf{u}) = T(\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix} = \begin{bmatrix} 1cu_1 + 2cu_2 \\ 1cu_2 \end{bmatrix} = c\begin{bmatrix} 1u_1 + 2u_2 \\ 1u_2 \end{bmatrix}$

$= cT(\mathbf{u})$

Hence, $T(c\mathbf{u}) = cT(\mathbf{u})$

Consequently, by (i) and (ii) T is a linear transformation.

Is the transformation F: $R^3 \rightarrow R^2$ defined by $F(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} x_1 x_2 \\ x_3^2 \end{bmatrix}$ a linear transformation?

Given $A = \begin{bmatrix} 2 & 4 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$, and $T{:}\mathrm{R}^3 \rightarrow \mathrm{R}^2$ defined by $T(\mathbf{x}) = A\mathbf{x}$.

Perform the following calculations:

$T(\mathbf{u}) =$

$T(\mathbf{v}) =$

$T(\mathbf{u} + \mathbf{v}) =$

$T(\mathbf{u}) + T(\mathbf{v}) =$

$T(3\mathbf{u}) =$

$3T(\mathbf{u}) =$

$T(4\mathbf{v}) =$

$4T(\mathbf{v}) =$

$T(3\mathbf{u} + 4\mathbf{v}) =$

Find all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $T(\mathbf{x}) = \mathbf{b}$.

Is T onto R^2? Is T 1-1?