MATRIX INVERSES

① In what sense is the matrix $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ special in the universe of 3×3 matrices?

② Given $A_{3\times 3}$ when can we find $C_{3\times 3}$ such that $AC = I_3$?

③ Suppose $C = [\bar{c}_1 \ \bar{c}_2 \ \bar{c}_3]$.

Note, $AC = [A\bar{c}_1 \vdots A\bar{c}_2 \vdots A\bar{c}_3]$.

Can we find $\bar{c}_1, \bar{c}_2, \bar{c}_3$ such that

$A\bar{c}_1 = \bar{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $A\bar{c}_2 = \bar{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$; $A\bar{c}_3 = \bar{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$?

④ Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix}$. Find $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ such that $AC = I_3$.

Here we seek c_{ij}'s for $1 \le i,j \le 3$ such that

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix}\begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix}\begin{bmatrix} c_{12} \\ c_{22} \\ c_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix}\begin{bmatrix} c_{13} \\ c_{23} \\ c_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can solve all three linear systems at the same time!

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

Reduce to RREF and pick out the values of the nine c_{ij}'s.

(5) Check your solution to #4 here.

(6) Now calculate CA.