Suppose $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$.

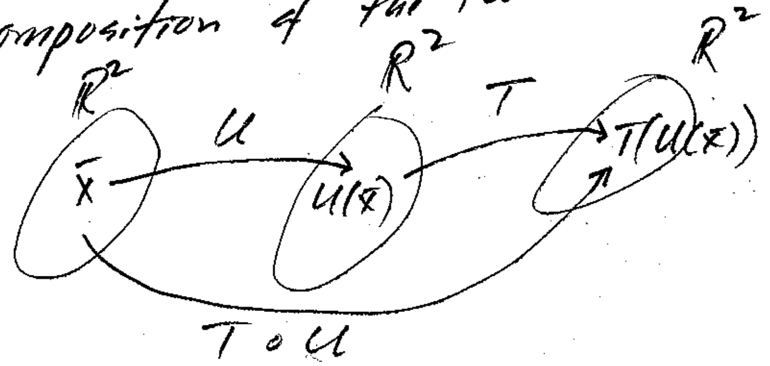
We define $T: \mathbb{R}^2 \to \mathbb{R}^2$ and $U: \mathbb{R}^2 \to \mathbb{R}^2$ as follows:

For $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$

$$T(\bar{x}) = A\bar{x} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$U(\bar{x}) = B\bar{x} = \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine the composition of the two functions $T \circ U(\bar{x})$.



$$T \circ U(\bar{x}) = T\left(U(\bar{x})\right) = T\left(\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\left(\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\left(x_1\begin{bmatrix} -1 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 5 \\ 2 \end{bmatrix}\right)$$

$$= x_1\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$= x_1\begin{bmatrix} 8 \\ 10 \end{bmatrix} + x_2\begin{bmatrix} 11 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 11 \\ 10 & 18 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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Suppose $\mathbb{A}_{m\times p} = [\bar{a}_1 \; \bar{a}_2 \cdots \bar{a}_p]$ and $\mathbb{B}_{p\times n} = [\bar{b}_1 \; \bar{b}_2 \cdots b_n]$

We define the matrix product $\mathbb{A}\mathbb{B}$ as follows:

$$(\star) \quad \mathbb{A}\mathbb{B} = [\mathbb{A}\bar{b}_1 \; \mathbb{A}\bar{b}_2 \cdots \mathbb{A}\bar{b}_n] = C_{m\times n}$$

We look at the (i,j)-entry of C

$$c_{ij} \text{ is the } i^{\text{th}} \text{ entry of } \mathbb{A}\bar{b}_j = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{pj} \end{bmatrix}$$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{ip} b_{pj}$$

$$(\star\star) \quad c_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$$

Calculate $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$ by both methods $(\star)$ and $(\star\star)$.