

## Session 9

How are the matrix equation

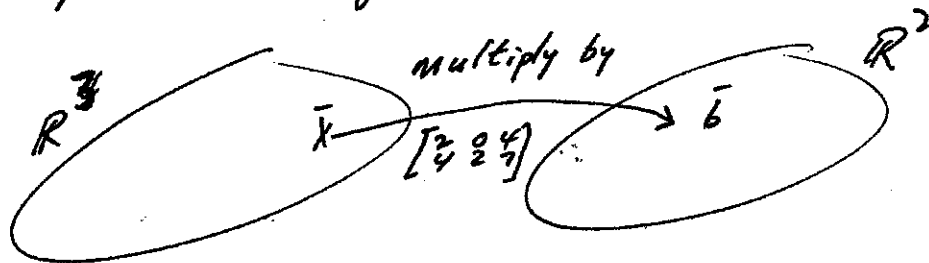
$$(1) \begin{bmatrix} 2 & 0 & 4 \\ 4 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \end{bmatrix}$$

and the vector equation

$$(2) x_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \end{bmatrix}$$

related?

Notice that we could think of the solutions to (1) and (2) above as those vectors  $\bar{x} \in \mathbb{R}^3$  that are "transformed" into the vector  $\begin{bmatrix} 10 \\ 18 \end{bmatrix}$  under multiplication by  $\begin{bmatrix} 2 & 0 & 4 \\ 4 & 2 & 7 \end{bmatrix}$ .



A transformation  $T$  (function, mapping) from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  is a rule that assigns to each vector  $\bar{x} \in \mathbb{R}^n$  a unique vector  $T(\bar{x}) \in \mathbb{R}^m$ .  $\mathbb{R}^n$  is called the domain of  $T$ ,  $\mathbb{R}^m$  is called the codomain of  $T$ . We denote this situation by  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ . For  $\bar{x} \in \mathbb{R}^n$  we call  $T(\bar{x})$  the image of  $\bar{x}$  (under  $T$ ). The set of all images  $T(\bar{x})$  is called the range of  $T$ .