Preliminaries

Axioms and Properties for the set R of Real Numbers

$a, b, c, d \in R$

I. Equality Axioms

1.	$a = a$	reflexive property
2.	If $a = b$, then $b = a$.	symmetric property
3.	If $a = b$ and $b = c$, then $a = c$.	transitive property
4.	If $a = b$, then "a" may be replaced by "b" or "b" may be repalced by "a" in any statement without affecting the truth of falsity of the statement.	substitution property

II. Order Axioms

1.	If $a, b \in R$, exactly one of the following is true: $a < b$, $a = b$, $a > b$	trichotomy property
2.	If $a < b$ and $b < c$, then $a < c$.	transitive property
3.	If $a > 0$ and $b > 0$, then $a + b > 0$ and $ab > 0$.	

III. Completeness Axiom

Each real number can be associated with a unique point on a number line and each point on a number line can be associated with a unique real number.

IV. Arithmetical Axioms

1.	$a + b \in R$	closure properety
2.	$a + b = b + a$	commutative property
3.	$(a + b) + c = a + (b + c)$	associative property
4.	$a + 0 = 0 + a = a$	additive identity
5.	$a + (-a) = (-a) + a$	additive inverse
6.	$ab \in R$	closure property
7.	$ab = ba$	commutative property
8.	$(ab)c = a(bc)$	associative property
9.	$a(1) = (1)a = a$	multiplicative identity
10.	$a(1/a) = (1/a)a = 1, a = 0$	multiplicative inverse
11.	$a(b + c) = ab + ac$	distributive property

V. Archimedian Axiom

For any $a, b \in R$, with $a > 0$, there exists an integer N such that $Na > b$.

VI. Density Property

For any $a, b \in R$, with $a < b$, there exists $c \in R$ such that $a < c < b$.

VII. Arithmetical Properties

1.	If $a = b$, then $a + c = b + c$	addition property
2.	If $a = b$, then $ac = bc$	multiplication property
3.	$a(0) = 0$	
4.	If $ab = 0$, then $a = 0$ or $b = 0$	
5.	If $a < b$, then $a + c < b + c$	addition property
6.	If $a < b$ and $c > 0$, then $ac < bc$ If $a < b$ and $c < 0$, then $ac > bc$	multiplication property
7.	$a - b = a + (-b)$	
8.	$a/b = a(1/b)$	

VIII. Absolute Value and Distance

1. $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a \leq 0 \end{cases}$
2. The distance between a and b is $|a - b|$.
3. $\{ x: \ b \leq |x - a| \leq c \}$