

Preliminaries
 Axioms and Properties for the set \mathbb{R} of Real Numbers
 $a, b, c, d \in \mathbb{R}$

I. Equality Axioms

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| 1. | $a = a$ | reflexive property |
| 2. | If $a = b$, then $b = a$. | symmetric property |
| 3. | If $a = b$ and $b = c$, then $a = c$. | transitive property |
| 4. | If $a = b$, then "a" may be replaced by "b" or "b" may be replaced by "a" in any statement without affecting the truth or falsity of the statement. | substitution property |

II. Order Axioms

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| 1. | If $a, b \in \mathbb{R}$, exactly one of the following is true:
$a < b$, $a = b$, $a > b$ | trichotomy property |
| 2. | If $a < b$ and $b < c$, then $a < c$. | transitive property |
| 3. | If $a > 0$ and $b > 0$, then
$a + b > 0$ and $ab > 0$. | |

III. Completeness Axiom

Each real number can be associated with a unique point on a number line and each point on a number line can be associated with a unique real number.

IV. Arithmetical Axioms

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|-----|---------------------------------|-------------------------|
| 1. | $a + b \in \mathbb{R}$ | closure property |
| 2. | $a + b = b + a$ | commutative property |
| 3. | $(a + b) + c = a + (b + c)$ | associative property |
| 4. | $a + 0 = 0 + a = a$ | additive identity |
| 5. | $a + (-a) = (-a) + a$ | additive inverse |
| 6. | $ab \in \mathbb{R}$ | closure property |
| 7. | $ab = ba$ | commutative property |
| 8. | $(ab)c = a(bc)$ | associative property |
| 9. | $a(1) = (1)a = a$ | multiplicative identity |
| 10. | $a(1/a) = (1/a)a = 1, a \neq 0$ | multiplicative inverse |
| 11. | $a(b + c) = ab + ac$ | distributive property |

V. Archimedean Axiom

For any $a, b \in \mathbb{R}$, with $a > 0$, there exists an integer N such that $Na > b$.

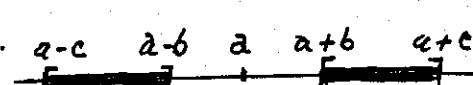
VI. Density Property

For any $a, b \in \mathbb{R}$, with $a < b$, there exists $c \in \mathbb{R}$ such that $a < c < b$.

VII. Arithmetical Properties

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|----|------------------------------------------------------------------------------------|-------------------------|
| 1. | If $a = b$, then $a + c = b + c$ | addition property |
| 2. | If $a = b$, then $ac = bc$ | multiplication property |
| 3. | $a(0) = 0$ | |
| 4. | If $ab = 0$, then $a = 0$ or $b = 0$ | |
| 5. | If $a < b$, then $a + c < b + c$ | addition property |
| 6. | If $a < b$ and $c > 0$, then $ac < bc$
If $a < b$ and $c < 0$, then $ac > bc$ | multiplication property |
| 7. | $a - b = a + (-b)$ | |
| 8. | $a/b = a(1/b)$ | |

VIII. Absolute Value and Distance

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|----|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| 1. | $ a = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$ | |
| 2. | The <u>distance</u> between a and b is $ a - b $. | |
| 3. | $\{ x : b \leq x - a \leq c \}$ |  |