Some Practice Exercises

1. Suppose $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -5 & -2 \\ 1 & 3 & -2 \end{bmatrix}$

   a) Describe Col A

   b) Describe Nul A

   c) Specify a basis for Col A

   d) Specify a basis for Nul A.

2. Determine whether the set $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} \right\}$ is linearly independent or linearly dependent. If the set is linearly dependent, write one of the vectors as a linear combination of the others.

3. Is $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1x_1 + 2x_2 - x_3 \\ 3x_2 + 6x_3 \end{bmatrix}$ which maps R^3 into R^2 a linear transformation? If so, specify the standard matrix for T. Is T onto R^2? Is T one-one?

4. Consider the mapping $T : R^3 \to R^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Does T^{-1} exist? If so, find a rule for T^{-1}; otherwise justify your answer.

5. For T defined as in #4 find $\bar{x} \in R^3$ such that $T(\bar{x}) = \begin{bmatrix} 2 \\ 11 \\ 25 \end{bmatrix}$.

Practice Exercises

Suppose $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 7 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix} \right\}$

1. Explain how you can tell that $\mathcal{B}$ is a set of linearly inpendent vectors and that $\mathcal{C}$ is not.

2. Explain how you can tell that span $\mathcal{B} = \mathbb{R}^3$ but span $\mathcal{C} \neq \mathbb{R}^3$.

3. Explain why $\mathcal{B}$ is a basis for $\mathbb{R}^3$ and $\mathcal{C}$ is not.

4. Suppose $\vec{x} = \begin{bmatrix} 10 \\ 8 \\ 27 \end{bmatrix}$. Find $[\vec{x}]_{\mathcal{B}}$.