Linear Algebra 10/26

(1) If $A = \begin{bmatrix} 2 & 6 & 0 & 4 & 0 \\ 0 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 & 5 \end{bmatrix}$ specify a basis for Col $A$.

dim Col $A$ = ___

rank $A$ = ___

dim Nul $A$ = ___

Find and specify a basis for Nul $A$.

(2) Which of the following sets, if any, are subspaces of $\mathbb{P}_4$?

a). All polynomials of the form $\bar{p}(t) = at^3$, where $a \in \mathbb{R}$.

b) All polynomals of degree at most 3 with integers as coefficients.

c) All polynomials in $\mathbb{P}_4$ such that $\bar{p}(0) = 0$.

d) All polynomials in the set Span$\{5, 2t, 6t^2\}$.

(3) Suppose $T: \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(\bar{u}) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 4 & -2 \end{bmatrix} \bar{u}$ for $\bar{u} \in \mathbb{R}^2$.

a) Determine the range of $T$.

b) Determine the kernel of $T$

The Spanning Set Theorem Let $S = \{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_p\}$ be a set of vectors in a vector space $V$ and let $H = \text{span}\{\bar{v}_1, \dots, \bar{v}_p\}$.

a) If one of the vectors in $S$, say $\bar{v}_k$, is a linear combination of the remaining vectors in $S$, then the set formed from $S$ by removing $\bar{v}_k$ still spans $H$.

b) If $H \neq \{\bar{0}\}$, some subset of $S$ is a basis for $H$.

## Unique Representation Theorem

Let $\mathcal{B} = \{\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n\}$ be a basis for a vector space $V$. Then for each $\bar{x} \in V$, there exists a unique set of scalars $c_1, c_2, \dots, c_n$ such that

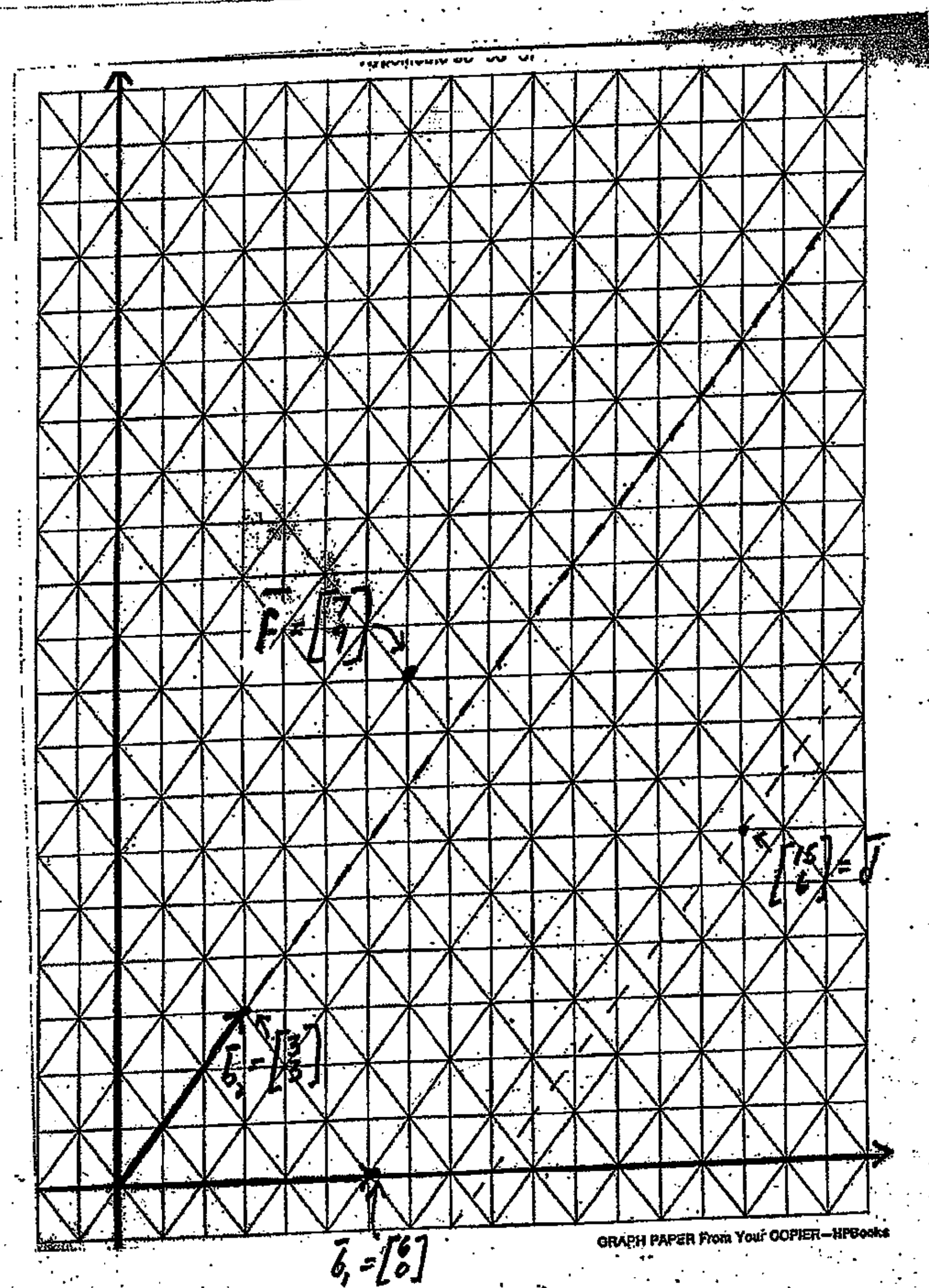
$$\bar{x} = c_1\bar{b}_1 + c_2\bar{b}_2 + \cdots + c_n\bar{b}_n.$$

## Coordinates

Suppose $\mathcal{B} = \{\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n\}$ is a basis for a vector space $V$ and $\bar{x} \in V$. The coordinates of $\bar{x}$ relative to the basis $\mathcal{B}$ (or the $\mathcal{B}$-coordinates of $\bar{x}$) are the weights $c_1, c_2, \dots, c_n$ such that $\bar{x} = c_1\bar{b}_1 + c_2\bar{b}_2 + \cdots + c_n\bar{b}_n$.

We denote this by $[\bar{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$.

(4) $\mathcal{B} = \{5, 2t, 6t^2\}$ is a basis for $P_2$. Find the $\mathcal{B}$-coordinates of $\bar{p}(t) = 10 + 6t + 30t^2$. That is, find $[\bar{p}(t)]_{\mathcal{B}}$.




Let $\mathcal{B} = \{\bar{b}_1, \bar{b}_2\}$ where $\bar{b}_1 = \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \bar{b}_2 = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

$\bar{d} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}, \ \bar{f} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

Find $[\bar{d}]_B$ and $[\bar{f}]_B$