Linear Algebra In-Class Exercises

Suppose $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p} \in \mathbf{R^n}$ and $c_1, c_2, \ldots, c_p \in \mathbf{R}$, a vector defined by

$$\mathbf{y} = c_1\mathbf{v}_1 + c_2\mathbf{v_2} + \ldots + c_p\mathbf{v_p}$$

is called a ***linear combination*** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ with weights $c_1, c_2, \ldots, c_p$.

Suppose $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p} \in \mathbf{R^n}$, then the set of all linear combinations of $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$ is called the ***subset of R^n spanned*** **(or *generated*) *by*** $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$. We denote this subset by Span$\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}\}$.

Exercises:

a. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 11 \\ 3 \\ 25 \end{bmatrix}$.

Find c_1, c_2, and c_2 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$.
Can $\mathbf{b}$ be written as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$?
Is $\mathbf{b}$ in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

b. Solve:
$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 11 \\ x_3 &= 3 \\ 2x_1 + 4x_2 + 5x_3 &= 25 \end{aligned}$$

c. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 11 \\ 3 \\ 28 \end{bmatrix}$.

Find c_1, c_2, and c_2 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{c}$.

d. Is $\mathbf{c}$ in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

e. Give a geometric description of Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

f. How many vectors are in Span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

g. Prove: For all vectors $\mathbf{u}, \mathbf{v} \in R^n$ and scalars c, $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$.