Special Matrices and Matrix Operations

If A is an *m x n* matrix, the entry in the i^{th} row and j^{th} column of A is denoted by a_{ij} and is called the *(i,j)*-entry of A.

The columns of A are vectors in R^m and are denoted by $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_n}$.

We sometimes write $A = [\mathbf{a_1} \ \mathbf{a_2} \ \ldots \ \mathbf{a_n}]$.

The *diagonal entries* in an *m x n* matrix $A = [a_{ij}]$ are $a_{11}, a_{22}, a_{33}, \ldots$ and they form the *main diagonal* of A. A *diagonal matrix* is a square matrix whose nondiagonal entries are all zero. An *m x n* matrix whose entries are all zero is a *zero matrix*.

$$K = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \qquad J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two matrices A and B are equal iff they are of the same size and their corresponding entries are equal.

If A and B are both *m x n* matrices, then the *sum* A + B is the *m x n* matrix C whose *(i,j)*-entry is defined by $c_{ij} = a_{ij} + b_{ij}$.

Suppose $A = \begin{bmatrix} 3 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 & 5 \\ 0 & 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$

A + B

A + C

If A is an *m x n* matrix and r is a scalar, then the scalar multiple rA is the *m x n* matrix whose *(i,j)*-entry is ra_{ij} . We define –A to mean (-1)A, and we write A – B in place of A + (-1)B.

Suppose A and B are defined as above.

(-2)B =

A – 2B =

We have already defined matrix multiplication. Supposing that A, B, C, and I are defined as above try to calculate each of the following:

CB

BC

BI