Subspaces of $\mathbf{R}^n$

1. A *subspace* of R^n is any set H in R^n that has three properties:
   a. The zero vector is in H.
   b. For each $\mathbf{u}$ and $\mathbf{v}$ in H, the sum $\mathbf{u} + \mathbf{v}$ is in H.
   c. For each $\mathbf{u}$ in H and each scalar c, the vector $c\mathbf{u}$ is in H.

   Suppose $K = \{\mathbf{v} \in R^2 : \mathbf{v} = k\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ for some scalar k $\}$. Is K a subspace of R^2?

   Suppose $J = \{\mathbf{v} \in R^2 : \mathbf{v} = \begin{bmatrix} k \\ 4 \end{bmatrix}$ for some scalar k $\}$. Is J a subspace of R^2?

2. The *column space* of a matrix A, denoted by *Col* A is the set of all linear combinations of the columns of A.

   Claim: If A is *m x n*, then *Col* A is a subspace of R^m.

3. The *null space* of a matrix A, denoted by *Nul* A, is the set of all solutions to the equation A**x** = **0**.

   Claim: If A is *m x n*, then *Nul* A is a subspace of R^m.

4. If $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 2 \\ 0 & 2 & -4 & 4 \end{bmatrix}$, describe *Col* A and *Nul* A.