

A set of m linear equations in n variables is called a *system of linear equations* or a *linear system*.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

In the linear system above each a_{ij} and b_i will be numbers and each x_j will be a variable. ($1 \leq i \leq m$; $1 \leq j \leq n$)

A *solution* for a linear system is an ordered n -tuple (s_1, s_2, \dots, s_n) of numbers that makes each equation in the system true when the values s_1, s_2, \dots, s_n are substituted for x_1, x_2, \dots, x_n respectively.

The set of all possible solutions for a linear system is called the *solution set* for the system. Two linear systems are called *equivalent* if they have the same solutions set.

We solve linear systems by a systematic procedure that allows us to replace one system with an equivalent system that is easier to solve. The following three actions produce equivalent linear systems:

Replacement – Replace one equation by the sum of itself and a multiple of another equation ($kE_j + E_i \rightarrow E_i$)

Interchange – Interchange two rows ($E_j \leftrightarrow E_i$)

Scaling – Multiply all entries in a row by a nonzero constant ($kE_i \rightarrow E_i$)