MATH 306 Linear Algebra Name________________________________
Sample Test

I. **Definitions.** (Three points each) Write definitions of the following terms. Be sure to use complete sentences in each case.

1. Define: linearly independent

2. Define: linear transformation

3. Define: subspace of R^n

4. Define: basis of a subspace of R^n

II. **Computations.** (Two points each) Given the matrices defined below, perform the indicated computations if possible. If a particular computation cannot be performed, say so and explain why it cannot be performed.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & 2 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -1 & 5 \\ -2 & 4 \end{bmatrix}$$

5. **AB**

6. **BA**

7. $\mathbf{B}^{\mathbf{T}} + \mathbf{C}$

8. 5**C**

III. **Exercises** (Five points each)

9. Consider T: $\mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 5x3, 2x_2 + 6x_3)$

a. Specify the standard matrix for T.

b. Is T onto $\mathbf{R}^2$? Justify your answer.

c. Is T one-to-one? Justify your answer.

10. Show, step-by-step, how to employ the row reduction algorithm to find the inverse of the matrix below.

$$\begin{bmatrix} 2 & 0 & 4 \\ 4 & 1 & 8 \\ 0 & 2 & 1 \end{bmatrix}$$

11. Suppose the matrix **A** is defined as follows.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix}$$

a. Is $\mathbf{x} = \begin{bmatrix} 10 \\ 4 \\ 8 \end{bmatrix}$ in the column space of **A**? Justify your answer.

c. Specify a basis for the column space of **A**.

d. What is the dimension of Col A?

e. Characterize the vectors in the null space of **A** in parametric vector form.

f. Specify a basis for Nul **A**.

12. Consider the set $H = \left\{ \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : \; x_1 - 2x_2 = 1 \right\}$.

a. Sketch the graph of H in 2-space.

b. Name two vectors in H.

c. Is H a subspace of $\mathbf{R}^2$? Justify your answer.

13. The standard basis for R^2 is $E = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and $B = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix} \right\}$ is another basis for R^2.

If the standard coordinates for $\mathbf{x} \in R^2$ are $\begin{bmatrix} 6 \\ 20 \end{bmatrix}$, then determine $[\mathbf{x}]_B$.