Assignment #12

3.1 #6

$$\begin{vmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{vmatrix} = (-1)^2 5 \begin{vmatrix} 3 & -5 \\ -4 & 7 \end{vmatrix} + (-1)^3(-2)\begin{vmatrix} 0 & -5 \\ 2 & 7 \end{vmatrix} + (-1)^4 4 \begin{vmatrix} 0 & 3 \\ 2 & -4 \end{vmatrix}$$

$$= 5(21-20) + 2(10) + 4(-6)$$

So, $\begin{vmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{vmatrix} = 1$

3.1 #22

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \overset{kR_2+R_1 \to R_1}{\sim} \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$$

$$\det \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix} = ad + kcd - bc - kcd$$
$$= ad - bc$$
$$= \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the value of the determinant is not affected

5.1 #4

$$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -2+2\sqrt{2}+1 \\ -1+\sqrt{2}+4 \end{bmatrix} = \begin{bmatrix} -1+2\sqrt{2} \\ 3+\sqrt{2} \end{bmatrix} = (3+\sqrt{2})\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$$

Note $(3+\sqrt{2})(-1+\sqrt{2}) = -3 + 3\sqrt{2} - \sqrt{2} + 2 = -1 + 2\sqrt{2}$

So, $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector for $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ with eigenvalue $(3+\sqrt{2})$.

5.1 #10

We seek all $\bar{x} \in \mathbb{R}^2$ such that

$$\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} \bar{x} = 4\bar{x}$$

$$\left[\begin{array}{cc|c} 6 & -9 & 0 \\ 4 & -6 & 0 \end{array}\right] \overset{\text{rref}}{\sim} \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \text{ for } t \in \mathbb{R}$$

So, a basis for the eigenspace corresponding to the eigenvalue $\lambda = 4$ is $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$.

5.1 #16

We seek all $\bar{x}$ such that $A\bar{x} = 4\bar{x}$

$$\begin{bmatrix} -1 & 0 & 2 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & -1 & -3 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{ for } t, u \in \mathbb{R}$$

So, a basis for the eigenspace corresponding to the eigenvalue $\lambda = 4$ is

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

5.2 #4

$$\det(A-\lambda I) = \det\begin{bmatrix} 5-\lambda & -3 \\ -4 & 3-\lambda \end{bmatrix}$$

$$= (5-\lambda)(3-\lambda) - 12$$

$$= 15 - 8\lambda + \lambda^2 - 12$$

$$= \lambda^2 - 8\lambda + 3$$

So, the characteristic polynomial is $\lambda^2 - 8\lambda + 3$

$$\lambda = \frac{8 \pm \sqrt{64-12}}{2} = 4 \pm 2\sqrt{13}$$

The eigenvalues are $4 + 2\sqrt{13}$ and $4 - 2\sqrt{13}$

5.2 #12

$$\det(A-\lambda I) = \det\begin{bmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{bmatrix} = (-1)^6(2-\lambda)(-1-\lambda)(4-\lambda)$$

$$= -(\lambda-2)(\lambda-1)(\lambda-4)$$

So the characteristic polynomial is $-(\lambda-2)(\lambda-1)(\lambda-4)$

5.2 #18

$$\det(A-\lambda I) = \det\begin{bmatrix} 5-\lambda & -2 & 6 & -1 \\ 0 & 3-\lambda & h & 0 \\ 0 & 0 & 5-\lambda & 4 \\ 0 & 0 & 0 & 1-\lambda \end{bmatrix}.$$ So, we

can see that $\lambda = 5$ is an eigenvalue of multiplicity two. We solve the homogeneous system with augmented matrix

$$\begin{bmatrix} 0 & -2 & 6 & -1 & 0 \\ 0 & -2 & h & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{bmatrix}$$

If $h = 6$ we will have two free variables x_1 and x_3. So, the eigenspace associated with $\lambda = 5$ is two dimensional.