Assignment #5

2.1 #6

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}_{3\times 2}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}_{2\times 2}$$

$$\underset{3\times 2}{AB} = \left[A\begin{bmatrix} 1 \\ 2 \end{bmatrix} \;\vdots\; A\begin{bmatrix} 3 \\ -1 \end{bmatrix} \right]$$ by the definition

$$= \left[1\begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix} \;\vdots\; 3\begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix} - 1\begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 4-4 & 12+2 \\ -3+0 & -9+0 \\ 3+10 & 9-5 \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$ by the row-column rule

$$= \begin{bmatrix} 4\cdot 1 - 2\cdot 2 & 4\cdot 3 - 2(-1) \\ -3\cdot 1 + 0\cdot 2 & -3\cdot 3 + 0(-1) \\ 3\cdot 1 + 5\cdot 2 & 3\cdot 3 + 5(-1) \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

2.1 #10

$$AB = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

2.1 #12

$$\begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2.1 #18

If the first two columns of $B = [\bar{b}_1 \; \bar{b}_2 \; \dots]$ are equal and for a matrix A, AB is defined, then the first two columns of AB are equal because they are the same linear combination of the columns of A.

2.1 / #24

Suppose $AD = I_m$.

We claim that for any $\bar{b} \in R^m$, the equation $A\bar{x} = \bar{b}$ has a solution.

Let $\bar{x} = D\bar{b}$

$$A\bar{x} = A(D\bar{b}) = (AD)\bar{b} = I\bar{b} = \bar{b}.$$

So, $D\bar{b}$ is the solution we seek.

Hence $AD = I_m \Rightarrow A\bar{x} = \bar{b}$ is consistent for all $\bar{b} \in R^m$

2.4 / #28 If we take $\bar{u}, \bar{v} \in R^n$ then each can be considered to be an $n \times 1$ matrix (or an $1 \times n$ matrix). Then by Thm 3 of this section

$$(\bar{u}^T V)^T = \bar{V}^T \bar{u}^{TT} = \bar{V}^T \bar{u}$$

Similarly

$$(uv^T)^T = v^{TT} u^T = vu^T$$

2.1 #30

Prove: Suppose A is $m \times n$ and B is of size such that AB is defined (B's columns are in R^n) and r is any scalar. It follows that $r(AB) = (rA)B = A(rB)$.

Note that the (i,j)-entry of $r(AB)$ is

$$r\sum_{k=1}^{n} a_{ik}b_{kj} = \sum_{k=1}^{n} r(a_{ik}b_{kj})$$

The (i,j)-entry of $(rA)B$ is

$$\sum_{k=1}^{n}(ra_{ik})b_{kj} = \sum_{k=1}^{n} r(a_{ik}b_{kj}) = (i,j)\text{-entry of } r(AB)$$

The (i,j)-entry of $A(rB)$ is

$$\sum_{k=1}^{n} a_{ik}(rb_{kj}) = \sum_{k=1}^{n} r(a_{ik}b_{kj}) = (i,j)\text{-entry of } r(AB)$$

Since the (i,j)-entries of $r(AB)$, $(rA)B$, $A(rB)$ are the same, those matrix products are the same.