$\boxed{\dfrac{1.1}{\#28}}$ Suppose a, b, c, d are constants and $a \neq 0$, and suppose the system below is consistent for all possible values of f and g.

What can we say about the numbers a, b, c, and d?

The linear system is $\begin{cases} ax_1 + bx_2 = f \\ cx_1 + dx_2 = g \end{cases}$

Reducing the augmented matrix for the system we obtain

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \overset{\frac{1}{a}R_1 \to R_1}{\sim} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ c & d & g \end{bmatrix} ; \quad a \neq 0$$

$$\overset{-cR_1 + R_2 \to R_2}{\sim} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ 0 & -\frac{cb}{a} + d & -\frac{cf}{a} + g \end{bmatrix}$$

In order for the system to be consistent for all values of f and g we must have $-\frac{cb}{a} + d \neq 0$. That is, $\frac{-cb + ad}{a} \neq 0$.

Hence, our system will be consistent provided $ad - cb \neq 0$.

1.2 #8

Consider the linear system with augmented matrix

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \overset{-2R_1+R_2 \to R_2}{\sim} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{bmatrix}$$

$$\begin{matrix} -1R_2 \to R_2 \\ -4R_2+R_1 \to R_1 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

Our general solution is

$$\begin{cases} x_1 = -9 \\ x_2 = 4 \\ x_3 \text{ is free} \end{cases}$$

1.2 #10

Consider the linear system with augmented matrix

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \overset{-3R_1+R_2 \to R_2}{\sim} \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

$$1R_2+R_1 \to R_1 \quad \begin{bmatrix} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

Our general solution is

$$\begin{cases} x_1 = 2x_2 - 4 \\ x_2 \text{ is free} \\ x_3 = -7 \end{cases}$$

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$$\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad -1R_2 + R_1 \to R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 7 & 0 & 0 & -9 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, our general solution is

$$\begin{cases} x_1 = -9 - 7x_3 \\ x_2 = 2 + 6x_3 + 3x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \\ x_5 \text{ is } 0 \end{cases}$$

Ex 1.2 #34

In this case we seek $a_0, a_1, a_2, \ldots, a_5$ such that

$$a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 + a_4(0)^4 + a_5(0)^5 = 0$$
$$a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 + a_4(2)^4 + a_5(2)^5 = 2.90$$
$$a_0 + a_1(4) + a_2(4)^2 + a_3(4)^3 + a_4(4)^4 + a_5(4)^5 = 14.8$$
$$a_0 + a_1(6) + a_2(6)^2 + a_3(6)^3 + a_4(6)^4 + a_5(6)^5 = 39.6$$
$$a_0 + a_1(8) + a_2(8)^2 + a_3(8)^3 + a_4(8)^4 + a_5(8)^5 = 74.3$$
$$a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3 + a_4(10)^4 + a_5(10)^5 = 119$$

The augmented matrix for the system is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 16 & 32 & 2.9 \\ 1 & 4 & 16 & 64 & 256 & 1024 & 14.8 \\ 1 & 6 & 36 & 216 & 1296 & 7776 & 39.6 \\ 1 & 8 & 64 & 512 & 4096 & 32768 & 74.3 \\ 1 & 10 & 100 & 1000 & 10{,}000 & 100{,}000 & 119 \end{bmatrix} \overset{\text{rref}}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1.7125 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1.1948 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0.6615 \\ 0 & 0 & 0 & 0 & 1 & 0 & -0.0701 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.0026 \end{bmatrix}$$

So, our interpolating polynomial is

✓ $p(t) \approx 1.7125t - 1.1948t^2 + 0.6615t^3 - 0.0701t^4 + 0.0026t^5$

If a projectile is traveling 750 ft/sec we evaluate p at $t = 7.5$ to estimate the force on the projectile.

✓ $p(7.5) \approx 64.605$. So, the force in this instance will be about 64.6 pounds.

Suppose we try to find an interpolating polynomial of degree less than 5 to exactly fit the data?

We consider the case of a cubic polynomial in this case we seek a_0, a_1, a_2, a_3 such that

$$a_0 + a_1(0)^1 + a_2(0)^2 + a_3(0)^3 = 0$$
$$a_0 + a_1(2)^1 + a_2(2)^2 + a_3(2)^3 = 2.90$$
$$a_0 + a_1(4)^1 + a_2(4)^2 + a_3(4)^3 = 14.8$$
$$a_0 + a_1(6)^1 + a_2(6)^2 + a_3(6)^3 = 39.6$$
$$a_0 + a_1(8)^1 + a_2(8)^2 + a_3(8)^3 = 74.3$$
$$a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3 = 119$$

The augmented matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 2.9 \\ 1 & 4 & 16 & 64 & 14.8 \\ 1 & 6 & 36 & 216 & 39.6 \\ 1 & 8 & 64 & 512 & 74.3 \\ 1 & 10 & 100 & 1000 & 119 \end{bmatrix} \overset{rref}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

✓ So our linear system is inconsistent and no such polynomial exists.