Ex 1.3

28. A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each ton of A burned, the plant produces 27.6 million Btu of heat, 3100 g of sulfur dioxide and 250 g of particulate matter. For each ton of B burned, the plant produces 30.2 million Btu, 6400 g of sulfur dioxide, and 360 g of particulate matter.

a. How much heat does the steam plant produce when it burns x_1 tons of A and x_2 tons of B?

The steam plant produces $27.6x_1 + 30.2x_2$ million Btu of heat when it burns x_1 tons of A and x_2 tons of B.

b. The linear combination of two vectors that would express the output of heat, sulfur dioxide, and particulate matter from the steam plant when it burns x_1 tons of A and x_2 tons of B would be the sum of $x_1\vec{A} + x_2\vec{B}$. This equation would look like:

$$x_1\begin{bmatrix}27.6\\3100\\250\end{bmatrix} + x_2\begin{bmatrix}30.2\\6400\\360\end{bmatrix}$$

c. In order to determine how many tons of each type of coal the steam plant must have burned to produce 162 million Btu, 23,610 g of sulfur dioxide, and 1623 g of particulate matter, we must find values for x_1 and x_2 that satisfy: $27.6x_1 + 30.2x_2 = 162$

$3100x_1 + 6400x_2 = 23610$

$250x_1 + 360x_2 = 1623$

To find this, we must solve the matrix:

$$\begin{bmatrix}27.6 & 30.2 & 162\\3100 & 6400 & 23610\\250 & 360 & 1623\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 3.9\\0 & 1 & 1.8\\0 & 0 & 0\end{bmatrix}$$

By reducing this matrix, we find that $x_1 = 3.9$ and $x_2 = 1.8$. Therefore, we can conclude that in order for the steam plant to produce 162 million Btu, 23,610 g of sulfur dioxide, and 1623 g of particulate matter, it must have burned 3.9 tons of anthracite coal and 1.8 tons of bituminous coal.