1.6 #12

A) Following example #2, I must first make a In and Out chart:

Intersection	In = Out
A	$x_1 = x_3 + 40 + x_4$
B	$200 = x_1 + x_2$
C	$x_2 + x_3 = 100 + x_5$
D	$x_4 + x_5 = 60$

I then swith the variable ~~to the left to set up~~ a linear system.

$$\begin{aligned} x_1 \quad - x_3 - x_4 &= 40 \\ x_1 + x_2 \qquad &= 200 \\ x_2 + x_3 \quad - x_5 &= 100 \\ x_4 + x_5 &= 60 \end{aligned}$$

Setting this up as an augmented matrix and row reducing I come to:

B) Following the same idea from A, I have the linear system:

$$\begin{aligned} x_1 \quad - x_3 \qquad &= 40 \\ x_1 + x_2 \qquad &= 200 \\ x_2 + x_3 \quad - x_5 &= 100 \\ x_5 &= 60 \end{aligned}$$

Once again putting this into an augmented matrix and reducing I get:

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 0 & 1 & 60 \end{array}\right] \xrightarrow{x_4 = 0} \text{RREF} \sim \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 40 \\ 0 & 1 & 1 & 0 & 0 & 160 \\ 0 & 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The general traffic flow again is:

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 = \text{Free} \\ x_4 = 0 \text{ (closed)} \\ x_5 = 60 \end{cases}$$

C.) When $x_4 = 0$, the minimum value of x_1 is $40 + x_3$. Since x_3 is free, it can be 0. This means assuming x_3 cannot be negative, x_1's minimum value is 40.