Section 1.7 Exercises

6. Determine if the columns of the matrix form a linearly independent set.

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

To study $A\bar{x}=\bar{0}$, row reduce the augmented matrix to row reduced echelon form.

$$\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \overset{R_3 \leftrightarrow R_1}{\sim} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ -4 & -3 & 0 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \underset{\substack{-5R_1+R_4 \to R_4 \\ -R_2 \to R_2}}{\overset{4R_1+R_3 \to R_3}{\sim}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & -3 & 12 & 0 \\ 0 & 4 & -9 & 0 \end{bmatrix}$$

$$\underset{-4R_2+R_4 \to R_4}{\overset{3R_2+R_3 \to R_3}{\sim}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix} \overset{\frac{1}{7}R_4 \to R_4}{\sim} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underset{4R_4+R_2 \to R_2}{\overset{-3R_4+R_1 \to R_1}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The are 3 basic variables and no free variables so the equation $A\bar{x}=\bar{0}$ has only the trivial solution and therefore the columns of the matrix A are linearly independent.